Homework 7
Instructor: Tyler Simon

Problem 1:
A computer system considers a string of decimal digits a valid codeword if it contains an even number of the digit 0. For instance, 1230407869 is valid, whereas 120987045608 is invalid. Let $a_n$ be the number of valid n-digit codewords.
Find a recurrence relation for $a_n$.
Note that $a_1 = 9$ since there are 10 one-digit strings, and only one is invalid, the "0" string.

Problem 2:
Suppose that the number of bacteria in a colony triples every hour.
• Set up a recurrence relation for the number of bacteria after $n$ hours have elapsed.
• If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

Problem 3:
In each question below, answer the following: Is $A \subseteq B$?, Is $B \subseteq A$? Is either $A$ or $B$ a proper subset of the other?
(1) $A = \{2, \{2\}, (\sqrt{2}^2)\}, B = \{2, \{2\}, \{\{2\}\}\}$
(2) $A = \{\{1, 2\}, \{2, 3\}\}, B = \{1, 2, 3\}$
(3) $A = \{\sqrt{16}, \{4\}\}, B = \{4\}$