Homework 3

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Problem 1:
Let $P(x)$ be the statement "$x$ spends more than five hours every weekday in class," where the domain for $x$ is the set of students in the class. Express each of the following quantifications in English.
(a.) $\exists x P(x)$
(b.) $\forall x P(x)$
(c.) $\exists x \neg P(x)$
(d.) $\forall x \neg P(x)$

Problem 2:
Translate the following statement into english, where $C(x)$ is "$x$ has a computer", $F(x, y)$ is "$x$ and $y$ are friends" and the domain for both $x$ and $y$ is the set of all students at UMBC.
$$\forall x (C(x) \lor \exists y (C(y) \land F(x, y)))$$

Problem 3:
For the following statements, the domain consists of all nonzero integers. Determine the truth value for each statement.

(a.) $\exists x \exists y [xy = 1]$
(b.) $\exists x \forall y [xy = 1]$
(c.) $\forall x \exists y [xy = 1]$
(d.) $\exists x \exists y [(2x + y = 5) \land (x - 3y = -8)]$
(e.) $\exists x \exists y [(3x - y = 7) \land (2x + 4y = 3)]$

Problem 4: Reorder the following numbered premises to make it clear that the conclusion follows logically from the premises and state the valid conclusion that can be drawn. (It may be helpful to rewrite some of the statements in "if-then" form and to replace some statements by their contrapositives.)
1.) No birds except ostriches are at least 9 feet tall.
2.) There are no birds in this aviary that belong to anyone but me.
3.) No ostrich lives on mince pieces.
4.) I have no birds less than 9 feet high.