FFT Application Examples and Implementation
FFT Example 1: Signal Sparsity in time Frequency Domain
FFT Example 2: Seizure Detection Problem

- Electrical signals can be detected by EEG signals before or just at the start of clinical symptoms
  - The ability to detect can be used to warn the patient or alert caregiver

Chandler et al
BioCass 2011
FFT Example 2: For Seizure, Frequency and Time domain analysis

• “We [doctors] detect seizures by looking at the evolution of frequency and amplitude in EEG” Dr. Jennifer Hopp, Epilepsy Center, UMMC.
A wearable solution for Multi-physiological signal processing

- Use FFT and filtering
FFT N-point implementation from FFT-2

- FFT N where N is a power of 2.
  - First calculate the Twiddle factors for half of N ($\frac{N}{2}$).
  - Construct the diagram of FFT N.
  - Construct a 2-point FFT.
  - Build the FFT N using the 2-point FFT ad diagram.

1) There are $\log_2 N$ stages.
2) There are $\frac{N}{2}$ of 2-point FFT per stage.
3) The order of inputs are in bit reverse.
4) For Twiddle factors, it shuffles between 2 samples, where $s = \frac{N}{2}$.
FFT-N Calculation from FFT-2 (Radix-2)

1. Compute Address (Input Index)
2. Read Data From Memory
3. Calculate Radix 2
4. Scaling and Write Data into Memory

Iteration = iteration + 1
An 8 Input Butterfly. Note, you double a 4 input butterfly, extend output lines, then connect the upper and lower butterflies together with diagonal lines.
FFT 16-Point

$x(0)$
$x(8)$
$x(4)$
$x(12)$
$x(2)$
$x(10)$
$x(6)$
$x(14)$
$x(1)$
$x(9)$
$x(5)$
$x(13)$
$x(3)$
$x(11)$
$x(7)$
$x(15)$

$X(0)$
$X(1)$
$X(2)$
$X(3)$
$X(4)$
$X(5)$
$X(6)$
$X(7)$
$X(8)$
$X(9)$
$X(10)$
$X(11)$
$X(12)$
$X(13)$
$X(14)$
$X(15)$

$W^0$
$W^4$
$W^0$
$W^4$
$W^0$
$W^2$
$W^4$
$W^6$
$W^0$
$W^1$
$W^2$
$W^3$
$W^4$
$W^5$
$W^6$
$W^7$
Twiddle factor Computation in Matlab

- Twiddle factor calculation in Matlab:

- Ex: N=8,
  - $W_0 = \exp(-2\pi \cdot 0 \cdot i/N) = 1$
  - $w_1 = \exp(-2\pi \cdot 1 \cdot i/N)$
  - ....
  - $W_7 = \exp(-2\pi \cdot 7 \cdot i/N)$
Radix-2 Butterfly Implementation

\[ A_{\text{Out}} = A + W \times B \]
\[ B_{\text{Out}} = A - W \times B \]

Where \( A = Ar + iAi, B = Br + iBi, W = Wr + iWi \)

So, \( A_{\text{Out}} = (Ar + WrBr - WiBi) + (Br + WrBi + WiBr) \)
\( B_{\text{Out}} = (Ar - WrBr + WiBi) + (Br - WrBi - WiBi) \)
FFT inputs bit reversal and memory addressing

- Bit reversal inputs example to FFT 8-point diagram

<table>
<thead>
<tr>
<th>Input</th>
<th>Bit Reversed</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>000</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>001</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>010</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>011</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>110</td>
<td>3</td>
</tr>
</tbody>
</table>

- Refer to for FFT-8 example (very useful) in slides:
  http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/osattari_ms.pdf
FFT Memory storage requirements for serial implementation

Memory Requirement:

1. Block RAM: IF overwriting per stage

   For FFT, you need N entries for Memory

   For 16-bit Real & Imag ⇒ N × 32 bits Mem

   \[ \log_2 16 = \text{4 stages} \]

   Per stage = \[ \frac{16}{2} = 8 \]
FFT Twiddle factor Memory

Memory Requirement

2. ROM/LUT for W factors

For FFT N: $e^{-j2\pi k/N}$, k is 0 ... $N/2$.

- No. of entries:

  FFT N: we need $N/2$ of real + $N/2$ of imag numbers.

- For W integer bits: $-1 < W < 1 \Rightarrow 2$ bits inter.
More details for FFT

• For more details refer to these documents:
  • http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/osattari_ms.pdf
  • http://www.csee.umbc.edu/~tinoosh/cmpe691/slides/Handout.fft1.diagrams.pdf
  • And FFT example from the book in the slides