Homework VII
Due: Wednesday, April 12, at the midterm

I. From the text:

§8.1: 6, 14, 19, 24, 35.
§8.2: 9, 38.
§8.3: 2, 12, 15.

II. In this last problem, we’ll find a closed-form expression for the sequence $a_n$ satisfying the recurrence

$$a_n = 3a_{n-1} - 2a_{n-2} + 1,$$  \hspace{1cm} (1)

with the initial conditions $a_0 = a_1 = 1$.

(a) The recurrence relation is not homogeneous, because of the pesky 1. However, $a_n$ does satisfy some homogeneous recurrences, and you can find one as follows. Use the recurrence (1) to solve for $a_{n-1}$ in terms of $a_{n-2}$ and $a_{n-3}$; this equation also has a pesky 1. Subtract the two equations, and the 1’s will cancel.

(b) Factor the characteristic polynomial of the homogeneous recurrence you found in (a). Show that the roots are 1 (twice) and 2 (once). Deduce that each of the three sequences 1, $n$, and $2^n$ satisfy the recurrence from (a).

(c) Find a linear combination of the three basic solutions from (b), which equals $a_n$. That is, find constants $A$, $B$ and $C$ such that

$$a_n = A \cdot 1 + B \cdot n + C \cdot 2^n.$$  

That’s all there is to it!