Homework 2, Due June 14th

You are required to solve TEN problems.
Problems 2, 3, 4, 5, 6 are not optional.
Solve any FIVE problems out of the rest.
Each problem carries 4 points.
Extra credit for extra problems solved.

1. Let $L \subseteq \Sigma^*$ be a regular language. Define the language $\text{Prefix}(L)$ as follows:
   \[
   \text{Prefix}(L) = \{w \in \Sigma^* : x = wy \text{ for some } x \in L, y \in \Sigma^* \}.
   \]
   Show that the language $\text{Prefix}(L)$ is also regular.

2. Construct DFAs equivalent to the corresponding NFAs given in the Figure 1.

   ![Figure 1](image)

3. Describe in English the sets denoted by the following regular expressions.
   (a) $(a \cup ba \cup bb)(a \cup b)^*$
   (b) $(a \cup b)^*a(a \cup b)(a \cup b)(a \cup b)$

4. Write regular expressions for each of the following languages over the alphabet \{0, 1\}.
   (a) $L_1 = \{w \mid w \text{ starts with 0 and has odd length, or } w \text{ starts with 1 and has even length} \}$
   (b) $L_2 = \{w \mid \text{every odd position of } w \text{ is a 1} \}$
5. Construct finite automata equivalent to the following regular expressions.
   
   (a) \((11 \cup 0^*)(00 \cup 1)^*\)
   
   (b) \(((00)^*(11)) \cup 01)^*\)

6. Construct regular expressions corresponding to the state diagrams given in the Figure 2.

![Figure 2](image)

Prove that the following languages are not regular using pumping lemma.

7. \(A_1 = \{0^n1^n2^n | n \geq 0\}\)

8. \(A_2 = \{x \in \{0, 1, 2\}^* | x = w2w, \text{ with } w \in \{0, 1\}^*\}\)

9. \(A_3 = \{a^naba^m | n, m \geq 1\}\)

10. \(A_4 = \{ww | w \in \{0, 1\}^*\}\)

11. \(A_5 = \text{Set of strings over } \{(, \}) \text{ in which the parentheses are paired. Some examples of strings in } A_5 \text{ are } (), ()(), (()), (()())\).

12. \(A_6 = \{ww^R | w \in \{0, 1\}^*\}. w^R \text{ is } w \text{ written backwards. For example, } (011)^R = 110\).