Viewing

CMSC 435/634
Coordinate System / Space

• Origin + Axes
• Reference frame
• Convert by matrix
  • $\vec{p}_{table} = TableFromPencil \vec{p}_{pencil}$
  • $\vec{p}_{room} = RoomFromTable TableFromPencil \vec{p}_{pencil}$
  • $\vec{p}_{room} = RoomFromPencil \vec{p}_{pencil}$
Spaces

• Object / Model
  • Logical coordinates for modeling
  • May have several more levels

• World
  • Common coordinates for everything

• View / Camera / Eye
  • eye/camera at (0,0,0), looking down Z (or -Z) axis
  • planes: left, right, top, bottom, near/hither, far/yon

• Normalized Device Coordinates (NDC) / Clip
  • Visible portion of scene from (-1,-1,-1) to (1,1,1)
  • Sometimes one or more components 0 to 1 instead of -1 to 1

• Raster / Pixel / Viewport
  • 0,0 to x-resolution, y-resolution

• Device / Screen
  • May translate to fit actual screen
Nesting

Room

- Desk
  - Student Book
  - Notebook
- Desk
  - Student Notebook
- Podium
- Board
- Laptop
- Eraser
Matrix Stack

• Remember transformation, return to it later
• Push a copy, modify the copy, pop
• Keep matrix and update matrix and inverse
• Push and pop both matrix and inverse together

```
transform (WorldFromRoom);  
push;                       
transform (RoomFromDesk);   
push;                       
transform (DeskFromStudent);  
pop;  push;                
transform (DeskFromBook);   
...                          
```
Spaces Perspective

Model → World / Model → View

- Model → World
  - All shading and rendering in World space
  - Transform all objects and lights
- Ray tracing implicitly does World → Raster
- Model → View
  - Serves just as well for single view
World→View

- Also called Viewing or Camera transform
- LookAt
  - \( \vec{from}, \vec{to}, \vec{up} \)
  - \[
  \begin{pmatrix}
  \vec{u} & \vec{v} & \vec{w} & \vec{from}
  \end{pmatrix}
  \]
- Roll / Pitch / Yaw
  - Translate to camera center, rotate around camera
  - \( R_z \ R_x \ R_y \ T \)
  - Can have gimbal lock
- Orbit
  - Rotate around object center, translate out
  - \( T \ R_z \ R_x \ R_y \)
  - Also can have gimbal lock
View→NDC

- Also called *Projection* transform
- Orthographic / Parallel
  - Translate & Scale to view volume
    \[
    \begin{bmatrix}
    \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\
    0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\
    0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\
    0 & 0 & 0 & 1
    \end{bmatrix}
    \]
- Perspective
  - More complicated...
NDC $\rightarrow$ Raster

- Also called *Viewport* transform
- $[-1, 1], [-1, 1], [-1, 1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]$
NDC → Raster

- Also called *Viewport* transform
- \([-1, 1], [-1, 1], [-1, 1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]\)

- Translate to \([0, 2], [0, 2], [0, 2]\)
NDC → Raster

• Also called Viewport transform
• $[-1, 1], [-1, 1], [-1, 1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]$
  
  • Translate to $[0, 2], [0, 2], [0, 2]$
  • Scale to $[0, n_x], [0, n_y], [0, n_z]$
NDC → Raster

- Also called Viewport transform
- $[-1, 1], [-1, 1], [-1, 1] \rightarrow [-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]$
  - Translate to $[0, 2], [0, 2], [0, 2]$
  - Scale to $[0, n_x], [0, n_y], [0, n_z]$
  - Translate to $[-\frac{1}{2}, n_x - \frac{1}{2}], [-\frac{1}{2}, n_y - \frac{1}{2}], [-\frac{1}{2}, n_z - \frac{1}{2}]$
NDC → Raster

- Also called Viewport transform
- $[-1, 1], [-1, 1], [-1, 1] \rightarrow \left[ -\frac{1}{2}, n_x - \frac{1}{2} \right], \left[ -\frac{1}{2}, n_y - \frac{1}{2} \right], \left[ -\frac{1}{2}, n_z - \frac{1}{2} \right]$
  - Translate to $[0, 2], [0, 2], [0, 2]$
  - Scale to $[0, n_x], [0, n_y], [0, n_z]$
  - Translate to $\left[ -\frac{1}{2}, n_x - \frac{1}{2} \right], \left[ -\frac{1}{2}, n_y - \frac{1}{2} \right], \left[ -\frac{1}{2}, n_z - \frac{1}{2} \right]$

$$\begin{bmatrix}
\frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\
0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\
0 & 0 & \frac{n_z}{2} & \frac{n_z - 1}{2} \\
0 & 0 & 0 & 1
\end{bmatrix}$$
Raster $\rightarrow$ Screen

- Usually just a translation
  - More complicated for tiled displays, domes, etc.
- Usually handled by windowing system
Perspective View Frustum

- Orthographic view volume is a rectangular volume
Perspective View Frustum

- Orthographic view volume is a rectangular volume
- Perspective is a truncated pyramid or \textit{frustum}
Perspective Transform

- Ray tracing
  - Given screen \((s^x, s^y)\), parameterize all points \(\vec{p}\)

\[ s^y / d = p_y / p_z \]
\[ s^y = dp_y / p_z \]
Perspective Transform

- Ray tracing
  - Given screen \((s^x, s^y)\), parameterize all points \(\vec{p}\)
- Perspective Transform
  - Given \(\vec{p}\), find \((s^x, s^y)\)

\[
\frac{s^y}{d} = \frac{p_y}{p_z}
\]

So
\[
s^y = dp_y / p_z
\]
Perspective Transform

- Ray tracing
  - Given screen \((s^x, s^y)\), parameterize all points \(\vec{p}\)
- Perspective Transform
  - Given \(\vec{p}\), find \((s^x, s^y)\)
  - Use similar triangles

\[
\frac{s^y}{d} = \frac{p^y}{p^z}
\]

So
\[
s^y = dp^y / p^z
\]
• Ray tracing
  • Given screen \((s^x, s^y)\), parameterize all points \(\vec{p}\)

• Perspective Transform
  • Given \(\vec{p}\), find \((s^x, s^y)\)
  • Use similar triangles
  • \(s^y / d = p^y / p^z\)
Perspective Transform

- Ray tracing
  - Given screen \((s^x, s^y)\), parameterize all points \(\vec{p}\)

- Perspective Transform
  - Given \(\vec{p}\), find \((s^x, s^y)\)
  - Use similar triangles
  - \(s^y / d = p^y / p^z\) So \(s^y = dp^y / p^z\)
Homogeneous Equations

- Same total degree for every term
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- $aX + bY + c = 0$
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- $aX + bY + c = 0$
  - $X = x/w$, $Y = y/w$
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- $aX + bY + c = 0$
  - $X = x/w$, $Y = y/w$
  - $ax/w + by/w + c = 0$
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
  \[aX + bY + c = 0\]
  - \[X = \frac{x}{w}, \ Y = \frac{y}{w}\]
  - \[ax/w + by/w + c = 0\]
  - \[\rightarrow ax + by + cw = 0\]
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- \( aX + bY + c = 0 \)
  - \( X = x/w, Y = y/w \)
  - \( a x/w + b y/w + c = 0 \)
  - \( \rightarrow a x + b y + c w = 0 \)
- \( a X^2 + bXY + c Y^2 + dX + eY + f = 0 \)
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- \( aX + bY + c = 0 \)
  - \( X = x/w, Y = y/w \)
  - \( ax/w + by/w + c = 0 \)
  - \( \rightarrow ax + by + cw = 0 \)
- \( aX^2 + bXY + cY^2 + dX + eY + f = 0 \)
  - \( X = x/w, Y = y/w \)
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- \( aX + bY + c = 0 \)
  - \( X = x/w, Y = y/w \)
  - \( ax/w + by/w + c = 0 \)
  - \( \rightarrow ax + by + cw = 0 \)
- \( aX^2 + bXY + cY^2 + dX + eY + f = 0 \)
  - \( X = x/w, Y = y/w \)
  - \( ax^2/w^2 + bxy/w^2 + cy^2/w^2 + dx/w + ey/w + f = 0 \)
Homogeneous Equations

- Same total degree for every term
- Introduce a new redundant variable
- $aX + bY + c = 0$
  - $X = x/w, Y = y/w$
  - $ax/w + by/w + c = 0$
  - $→ ax + by + cw = 0$
- $aX^2 + bXY + cY^2 + dX + eY + f = 0$
  - $X = x/w, Y = y/w$
  - $ax^2/w^2 + bxy/w^2 + cy^2/w^2 + dx/w + ey/w + f = 0$
  - $→ ax^2 + bxy + cy^2 + dxw + eyw + fw^2 = 0$
Homogeneous Coordinates

- Rather than \((x, y, z, 1)\), use \((x, y, z, w)\)
- Real 3D point is \((X, Y, Z) = (x/w, y/w, z/w)\)
- Can represent Perspective Transform as 4x4 matrix

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1/d & 0
\end{bmatrix}
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
p^z/d
\end{bmatrix}
\rightarrow 
\begin{bmatrix}
d & p^x/p^z \\
d & p^y/p^z \\
d & p^z/d \\
d
\end{bmatrix}
\]
Homogeneous Depth

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1/d & 0
\end{bmatrix}
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
p^z/d
\end{bmatrix} 
\rightarrow 
\begin{bmatrix}
d p^x/p^z \\
d p^y/p^z \\
d
\end{bmatrix}
\]

- Lose depth information
- Can’t get \( d p'^z/p^z = p^z \)
  - Plus \( x/z, y/z, z \) isn’t linear
- Use Projective Geometry
Projective Geometry

- If $x, y, z$ lie on a plane, $x/z, y/z, 1/z$ also lie on a plane
- $1/z$ is strictly ordered: if $z_1 < z_2$, then $1/z_1 > 1/z_2$
- New matrix:

$$\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
p^x \\
p^y \\
p^z \\
1
\end{bmatrix} = \begin{bmatrix}
p^x \\
p^y \\
1 \\
p^z
\end{bmatrix} \rightarrow \begin{bmatrix}
p^x/p^z \\
p^y/p^z \\
1/p^z
\end{bmatrix}$$
Getting Fancy

- Add scale & translate
  - Field of view
  - near/far range

\[
\begin{bmatrix}
a & 0 & 0 & 0 \\
0 & a & 0 & 0 \\
0 & 0 & b & c \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
1 \\
\end{bmatrix}
= 
\begin{bmatrix}
ap^x \\
ap^y \\
b \:^z + c \\
-^z \\
\end{bmatrix}
\rightarrow 
\begin{bmatrix}
-a \:^x / \:^z \\
-a \:^y / \:^z \\
-b - c / \:^z \\
\end{bmatrix}
\]

- \(a = \cotan(fieldOfView / 2)\)
- Solve for \(n \rightarrow -1\) and \(f \rightarrow 1\)
Getting Fancy

- Add scale & translate
  - Field of view
  - near/far range

\[
\begin{bmatrix}
a & 0 & 0 & 0 \\
0 & a & 0 & 0 \\
0 & 0 & b & c \\
0 & 0 & -1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
p^x \\
p^y \\
p^z \\
1 \\
\end{bmatrix}
= 
\begin{bmatrix}
ap^x \\
ap^y \\
b\,p^z + c \\
-p^z \\
\end{bmatrix}
\rightarrow 
\begin{bmatrix}
-a\,p^x/p^z \\
-a\,p^y/p^z \\
-b - c/p^z \\
\end{bmatrix}
\]

- \( a = \cotan\left(\frac{\text{fieldOfView}}{2}\right) \)
- Solve for \( n \rightarrow -1 \) and \( f \rightarrow 1 \)
  - \( b = \frac{(n + f)}{(n - f)} \)
  - \( c = \frac{(2 \, n \, f)}{(f - n)} \)
On Field of View

- Given image dimensions, set distance
  - Camera image sensor and focal length
- Given field of view angle in square window
- Non-square aspect ratio
  - Given horizontal (or vertical) field of view
  - Given diagonal field of view
- Off-center projection
  - Tiled displays
  - Head tracking