Illumination

- Effect of light on objects
- Mostly look just at intensity
  - Apply to each color channel independently
- Good for most objects
  - Not fluorescent
  - Not phosphorescent
Local vs. Global

- **Local**
  - Light sources shining directly on object

- **Global**
  - Lights bouncing from objects onto other objects
  - Ambient Illumination
    - Approximate global illumination as constant color
    - Typically $\sim 1\%$ of direct illumination
Bidirectional Reflectance Distribution Function
How much light reflects from $L_i$ to $L_o$

Surface normal
Incoming radiance
Outgoing radiance
Physically Plausible BRDF

- Positive
- Reciprocity
  - Same light from $L_i$ to $L_o$ as from $L_o$ to $L_i$
- Conservation of Energy
  - Don’t reflect more energy than comes in
Plotting BRDFs

- Polar plot of reflectance strength
  - For one view direction, showing light directions
  - For one light direction, showing view directions
- Reciprocity – same if you swap view and light
Rendering Equation

Integral of all Incoming Light

\[ L_o(\hat{v}) = \int_{\Omega(\hat{n})} L_i(\hat{l}) f_r(\hat{v}, \hat{l}) \hat{n} \cdot \hat{l} d\omega(\hat{l}) \]

Parts of this equation:

- \( L_o(\hat{v}) \): outgoing light in direction \( \hat{v} \)
- \( \Omega(\hat{n}) \): hemisphere above \( \hat{n} \) that can see this point
- \( L_i(\hat{l}) \): incoming light from direction \( \hat{l} \)
- \( f_r(\hat{v}, \hat{l}) \): BRDF from \( \hat{l} \) to \( \hat{v} \)
- \( \hat{n} \cdot \hat{l} d\omega(\hat{l}) \): projection of differential solid angle onto surface
Rendering Equation for Point Lights

Sum for Each Light

\[
L_o(\hat{v}) = \sum_i L_i f_r(\hat{v}, \hat{l}_i) \hat{n} \cdot \hat{l}_i
\]

Parts of this equation:

- \(L_o(\hat{v})\): outgoing light in direction \(\hat{v}\)
- \(i\): lights that can see this point (where \(\hat{n} \cdot \hat{l}_i > 0\))
- \(\hat{l}_i\): light direction to light \(i\)
- \(L_i\): incoming light for light \(i\)
- \(f_r(\hat{v}, \hat{l})\): BRDF from \(\hat{l}_i\) to \(\hat{v}\)
Results

- Integrating full environment
- Light at one point, black elsewhere
Decomposing BRDFs

- Decompose BRDF into convenient parts
- Typical breakdown:
  - Diffuse (view independent)
  - Specular (view dependent near reflection)
  - Others less common, often ignored (e.g. retro reflection)

\[
L_o(\hat{v}) = \sum_i L_i \left( f_d(\hat{v}, \hat{l}_i) + f_s(\hat{v}, \hat{l}_i) \right) \hat{n} \cdot \hat{l}_i
\]

\[
L_o(\hat{v}) = \sum_i L_i f_d(\hat{v}, \hat{l}_i) \hat{n} \cdot \hat{l}_i + \sum_i L_i f_s(\hat{v}, \hat{l}_i) \hat{n} \cdot \hat{l}_i
\]
Important directions

$\hat{n}$: Unit surface normal
Important directions

\( \hat{\nu} \): Unit vector from surface toward viewer
Important directions

\( \hat{l} \): Unit vector from surface toward light
Important directions

\[ \hat{R}_v = 2\hat{n}(\hat{n} \cdot \hat{v}) - \hat{v} : \text{Direction of mirror reflection of view} \]
Important directions

\[ \hat{R}_l = 2\hat{n}(\hat{n} \cdot \hat{l}) - \hat{l} \]: Direction of mirror reflection of light
Important directions

\[ \hat{h} = (\hat{v} + \hat{l}) / |\hat{v} + \hat{l}|: \text{ Normal direction that would reflect } \hat{v} \text{ to } \hat{l} \]
Diffuse

- Also called Lambertian or Matte
- Total reflectance: $\sum_i L_i Kd \hat{n} \cdot \hat{i}_i$
- BRDF: $Kd$
### Phong

- Strongest where $\hat{R}_i$ lines up with $\hat{v}$ or $\hat{R}_v$ lines up with $\hat{l}$
- Total reflectance: $\sum_i L_i K_s (\hat{R}_v \cdot \hat{l}_i)^e$
- Physically plausible version: $\sum_i L_i K_s (\hat{R}_v \cdot \hat{l}_i)^e \hat{n} \cdot \hat{l}$
  - With energy-conserving $K_s$
Specular Microfacets

- Imagine random mirrored microfacets

- Normal Distribution Function (NDF)
  - Probability facet has normal \( \hat{h} \)
  - Only facets to reflect \( \hat{l} \) to \( \hat{v} \)

- Proportion of light or view blocked (geometry term)
  - Blocked light = shadowing
  - Blocked view = masking

- Fresnel term
  - Reflection from non-metals is stronger at glancing angles
Cook-Torrance

- Beckmann Distribution = Gaussian distribution of slope
- Shadow/Mask based on symmetric V-shaped microfacets
- BRDF: \( D(\hat{n}, \hat{h}) \frac{G(\hat{n}, \hat{v}, \hat{l})}{4 \hat{n} \cdot \hat{v} \hat{n} \cdot \hat{l}} F(\hat{v}, \hat{l}) \),
- Total reflectance: \( \sum_i L_i Ks D(\hat{n}, \hat{h}_i) \frac{G(\hat{n}, \hat{v}, \hat{l}_i)}{4 \hat{n} \cdot \hat{v} \hat{n} \cdot \hat{l}} F(\hat{v}, \hat{l}_i) \hat{n} \cdot \hat{l} \)
Blinn-Phong

- Alternate formulation for Phong, similar behavior
- Strongest where $\hat{h}$ lines up with $\hat{n}$
  - Function of $\hat{h}$, behaves like NDF
- Total reflectance (original form): $\sum_i L_i K_s (\hat{n} \cdot \hat{h}_i)^\epsilon$
- As NDF: $D(\hat{n}, \hat{h}_i) = \frac{e+2}{2\pi} (\hat{n} \cdot \hat{h}_i)^\epsilon$
When to Compute

- **Gouraud Shading** = Compute per-vertex & interpolate
  - Lose sharp highlights
  - Subject to *Mach banding*
- **Phong Shading** = Interpolate normals & compute per-pixel
Phong Shading

- Phong shading can refer to lighting model or interpolation
- To save confusion:
  - Phong lighting
  - Phong interpolation