Comparative Analysis of Web Search and Ranking Algorithms

Mihir Kelkar

University of Maryland, Baltimore County. CMSC 676

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Overview of the Talk

• Brief History of Search
• Impact of Web Search Algorithm on our lives
• The Google Page Rank Algorithm
• The HITS Algorithm
• Facebook's Edge Rank Algorithm
• Reddit's Story Ranking Algorithm
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Brief History of Search

(a) Search in 300 BC
Brief History of Search

(b) Card Catalogs
Brief History of Search

(c) Archie
(d) Wandex
(e) Infoseek
Brief History of Search

(f) Altavista
(g) Yahoo Search
(h) Google Search
(i) Ask
(j) MSN
(k) Microsoft Bing
(1) Navigation is essentially using a search
Search Algorithms have a major impact on how you behave
Impact of Search Algorithms

Facebook's search feature actually modifies the stories that you see on top.
• PageRank is a "vote", by all the other pages on the Web, about how important a page is.
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The Google Page Rank Algorithm

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• PageRank is a "vote", by all the other pages on the Web, about how important a page is.
• The World Wide Web can be visualized as a highly interconnected graph with directed edges
• Not all pages on the internet are equally "important"
• The more important pages you "cite" your page’s content becomes that much more "credible and important"
A page links to various other pages, this forms a directed graph.
The Google Page Rank Algorithm

• Webpages are not equally "important".
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- An incoming link to page X stands as a vote of importance for page X
- www.stanford.edu has about 24000 incoming links whereas www.notdecided.com has just 1.
- Think of the incoming link as a sort of "commendation". So, an incoming link from different pages has different weights associated with it. The more important the source page, the more weight its outgoing link carries.
The Google Page Rank Algorithm

- \( PR(A) = (1 - d) + d \left( \frac{PR(T_1)}{C(T_1)} + \ldots + \frac{PR(T_n)}{C(T_n)} \right) \)

- \( PR(A) \) is the notation for page rank of A
- \( PR(T_i) \) - Page Rank of pages Ti which link to page A
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The Google Page Rank Algorithm

- $PR(A) = 0.5 + 0.5 \times PR(C)$
- $PR(B) = 0.5 + 0.5 \times (PR(A) / 2)$
- $PR(C) = 0.5 + 0.5 \times (PR(A) / 2 + PR(B))$
The Google Page Rank Algorithm

- Because of the size of the actual web, Google uses an approximative, iterative computation of PageRank values.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>PR(A)</th>
<th>PR(B)</th>
<th>PR(C)</th>
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<td>1</td>
<td>1</td>
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<td>10</td>
<td>1.07692305</td>
<td>0.76923076</td>
<td>1.15384615</td>
</tr>
</tbody>
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The Hyperlink Induced Topic Search Algorithm

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- An authority is a page that many hubs link to.
- A hub is a page that links to many authorities.
The Hyperlink Induced Topic Search Algorithm

(q) News Websites in the Chicago area represented as Hubs and Authorities
The Hyperlink Induced Topic Search Algorithm

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• The Authority score for a node is updated according to the authority update rule: \( \text{auth}(x) = \sum_{i=1}^{n} \text{hub}(i) \)

A k-step application of the Hub-Authority algorithm entails applying for k times first the Authority Update Rule and then the Hub Update Rule.

The values are normalized to make sure that they remain converging.
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The Hyperlink Induced Topic Search Algorithm

The Adjacency matrix for this graph can be represented as follows:

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
The Hyperlink Induced Topic Search Algorithm

\[
A = \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\text{ hence } A^t = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 1 & 0
\end{pmatrix}. 
\]

Also Assume that the initial hub vector is \( u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \).
The Hyperlink Induced Topic Search Algorithm

We compute the Authority weight vector as $A^t.u$

\[
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
2
\end{pmatrix}
\]

We compute the Hub weight vector as $A.A^t.u$

\[
\begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
0 \\
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This somewhat already corresponds with our intuition that node 3 must be authoritative.
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Facebook’s Edge Rank Algorithm

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• The entire userbase of social media websites can be visualized as a directed graphs with users as nodes and interactions between them as edges.

• The deciding factor about which stories should significantly appear on your news feed is how often you "interact" with the person who is the source/participant in the story.
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- Weight represented as $w$
- Time Decay: represented as $T_d$
Explanation of User Affinity: $u_e$

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- Affinity can however also be directly declared, by listing someone as directly related to you. Eg. A Brother, A parent or a spouse.
Explanation of Weight: $W$

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Explanation of Weight: $W$

- All kinds of interactions have different weights associated with them.
- An interaction like commenting has a higher weight over an interaction like simply liking the story since commenting needs the user to be more involved in the story in general.
- As a generalization, the more time consuming a method of interaction, the more weight it carries.
Explanation of Time Decay: $T_d$

- Each story has an associated "lifetime", the older it gets, the lower it gets ranked.
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- However, Time decay also considers time since last interaction between the two nodes.
- Thus for someone who logs in very irregularly, older stories still appear as Top Ranked stories. However for someone who logs in frequently, top ranked stories change faster.
Reddit’s Story Ranking Algorithm

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• The Ranking function for Reddit’s stories

\[ f(T_s, y, z) = \log(z) + \frac{yT_s}{45000}. \]
Reddit’s Story Ranking Algorithm

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  \[ f(T_s, y, z) = \log(z) + \frac{yT_s}{45000}. \]
The Ranking function uses Logarithm to weight the total number of upvotes. I believe that this is done so to make sure that the initial few votes count higher than the rest.
Reddit’s Story Ranking Algorithm

- 10 up, 0 down, submission: now
- 30 up, 0 down, submission: now
- 100 up, 0 down, submission: now
- 300 up, 0 down, submission: now
- 1000 up, 0 down, submission: now
- 3000 up, 0 down, submission: now
Reddit’s Story Ranking Algorithm

- Reddit is one of the few sites which have the brutal downvote button. Downvotes can significantly affect a story’s rank.
Reddit’s Story Ranking Algorithm

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The first 10 upvotes count as high as the next 100. E.g. a story that has 10 upvotes and a story that has 50 upvotes will have a similar ranking
Controversial stories that get similar amounts of upvotes and downvotes will get a low ranking compared to stories that mainly get upvotes
Sources and Citations

- The PageRank Citation Ranking: Bringing Order to the Web
- Authoritative Sources in a Hyperlinked Environment
- Reddit Engineering Blog