CMSC 441: Homework #6 Solutions

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Parag Namjoshi
Exercise 7.1–1
Show result of partition for array \( A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21] \).

**Solution:**
Pivot element is 21. Thus \( A \) remains the same after the PARTITION operation. \( A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21] \).

Exercise 7.1–4

Change the line 4 of PARTITION from \( A[j] \leq x \) to \( A[j] \geq x \).

Exercise 7.2–1

Use substitution method to prove that the recurrence \( T(n) = T(n-1) + \Theta(n) \) has the solution \( T(n) = \Theta(n^2) \).

**Solution:**
We guess that \( T(n) \leq O(n^2) \).

\[
T(n) \leq c_1(n-1)^2 + \Theta(n)
\leq c_1(n-1)^2 + c_0 n
\leq c_1(n^2 - 2n + 1) + c_0 n
\leq c_1 n^2 - (2c_1 - c_0) n + c_1
\leq c_1 n^2 \text{ for } n_0 \geq 1 \text{ and } c_0 > c_1
\]

Thus \( T(n) \in O(n^2) \). Similarly, we can prove that \( T(n) \in \Omega(n^2) \). Consequently, \( T(n) \in \Theta(n^2) \).

Exercise 7.4–1

Show that in the recurrence

\[
T(n) = \max_{0 \leq q \leq n-1} (T(q) + T(n-q-1)) + \Theta(n)
\]

\[
T(n) = \Omega(n^2)
\]

**Solution:**
We guess that \( T(n) \geq cn^2 \) for some constant \( c \). Substituting, we get

\[
T(n) \geq \max_{0 \leq q \leq n-1} (cq^2 + c(n-q-1)^2) + \Theta(n)
\]

\[
= c \cdot \max_{0 \leq q \leq n-1} (q^2 + (n-q-1)^2) + \Theta(n)
\]
The pure quadratic \( q^2 + (n - q - 1)^2 \) achieves its maximum at two end-points, 0, \( n - 1 \). We choose \( q = 0 \). Thus \( \max_{0 \leq q \leq n-1} (q^2 + (n - q - 1)^2) = (n - 1)^2 = n^2 - 2n + 1 \).

\[
T(n) \geq cn^2 - c(2n - 1) + \Theta(n) \\
\geq cn^2, \text{where } c \text{ is chosen to dominate } \Theta(n).
\]

Exercise 8.2–1

Step (a)
A: 6,0,2,0,1,3,4,6,1,3,2
C: 2,2,2,1,0,2

Step (b)
C: 2,4,6,8,9,9,11

Step (c)
B: -, -, -, -, 2, -, -, -, -
C: 2,4,5,8,9,9,11

Step (d)
B: -, -, -, -, 2, -, 3, -, -, -
C: 2,3,5,7,9,9,11

Step (e)
B: -, -, -, 1, -, 2, -, 3, -, -
C: 2,3,5,7,9,9,11

Step (f)
B: -, -, -, 1, -, 2, -, 3, -, 6
C: 2,3,5,7,9,10

Step (g)
B: -, -, 1, -, 2, -, 3, -, 6
C: 2,3,5,7,9,10

Step (h)
B: -, 1, 1, -, 2, 3, 3, 4, -, 6
C: 2,2,5,6,8,9,10

Step (i)
B: -, 1, 1, -, 2, 3, 3, 4, -, 6
C: 2,2,5,6,8,9,10

Step (j)
B: -, 0, 1, 1, -, 2, 3, 3, 4, -, 6
C: 1,2,5,6,8,9,10

Step (k)
B: -, 0, 1, 1, 2, 3, 3, 4, -, 6
C: 0,2,4,6,8,9,10

Step (l)
B: 0, 0, 1, 1, 2, 3, 3, 4, 6, 6
C: 0,2,4,6,8,9,9

Step (m)
B: 0, 0, 1, 1, 2, 3, 3, 4, 6, 6
C: 0,2,4,6,8,9,9
Exercise 8.3–1

Radix–sort on the list COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

COW  SEA  TAB  BAR
DOG  TEA  BAR  BIG
SEA  MOB  EAR  BOX
RUG  TAB  TAR  COW
ROW  DOG  SEA  DIG
MOB  RUG  TEA  DOG
BOX  DIG  DIG  EAR
TAB  BIG  BIG  FOX
BAR  BAR  MOB  MOB
EAR  EAR  DOG  NOW
TAR  TAR  COW  ROW
DIG  COW  ROW  RUG
BIG  ROW  NOW  SEA
TEA  NOW  BOX  TAB
NOW  BOX  FOX  TAR
FOX  FOX  RUG  TEA

Exercise 8.4–1

Illustrate the operation of Bucket-Sort on the array $A = [.79, .13, .16, .64, .39, .20, .89, .53, .71, .42]$. 

**Solution:**

| .13 | .16 |
| .20 |    |
| .39 | .20 |
| .42 | .64 |
| .53 | .71 |
| .64 | .79 |
| .71 | .89 |