Problem Set #2 Solutions

1. Rosen #1.6.4:

Each of the sets is a subset of itself. Aside from that, the only relations are $B \subseteq A$ and $C \subseteq D$.

2. Rosen #1.6.18:

a. The power set of every set includes at least the empty set, so the power set cannot be empty. Thus, the empty set $\emptyset$ cannot be the power set of any set.

b. This is the power set of $\{a\}$.

c. This set has three elements. The power set of any finite set must have $2^n$ elements, where $n$ is the number of elements in the original set. Hence, this set cannot be the power set of any set.

d. This is the power set of $\{a, b\}$.

3. Rosen #1.6.20:

By definition, it is all ordered pairs $(c, p)$, where $c$ is a course and $p$ is a professor.

4. Rosen #1.7.6:

a. $A \cup \emptyset = \{x | x \in A \lor x \in \emptyset\} = \{x | x \in A \lor F\} = \{x | x \in A\} = A$

b. $A \cap \emptyset = \{x | x \in A \land x \in \emptyset\} = \{x | x \in A \land F\} = \{x | F\} = \emptyset$

c. $A \cup A = \{x | x \in A \lor x \in A\} = \{x | x \in A\} = A$

d. $A \cap A = \{x | x \in A \land x \in A\} = \{x | x \in A\} = A$

e. $A - \emptyset = \{x | x \in A \land x \notin \emptyset\} = \{x | x \in A \land T\} = \{x | x \in A\} = A$

f. $A \cup U = \{x | x \in A \lor x \in U\} = \{x | x \in A \lor T\} = \{x | T\} = U$

g. $A \cap U = \{x | x \in A \land x \in U\} = \{x | x \in A \land T\} = \{x | x \in A\} = A$

h. $\emptyset - A = \{x | x \in \emptyset \land x \notin A\} = \{x | x \in F \land x \notin A\} = \{x | F\} = \emptyset$
5. Rosen #1.7.20

The Venn diagrams for the sets are below:

(a)  
(b)  
(c)  

6. Rosen #1.7.40

a. \((0, 0, 1, 1, 0, 0, 0, 0, 0, 0)\)

b. \((1, 0, 1, 0, 0, 1, 0, 0, 0, 1)\)

c. \((0, 1, 1, 1, 0, 0, 1, 1, 1, 0)\)

7. Rosen #1.8.6

a. The domain is \(\mathbb{Z}^+ \times \mathbb{Z}^+\) and the range is \(\mathbb{Z}^+\).

b. Since the largest decimal digit of a strictly positive integer cannot be 0, we have domain \(\mathbb{Z}^+\) and range \(\{1, 2, 3, 4, 5, 6, 7, 8, 9\}\).

c. The domain is the set of all bit strings. If we view this difference as being a signed difference (number of 1’s minus number of 0’s), then the range is \(\mathbb{Z}\). If we view it as the absolute value of this quantity, then the range is \(\mathbb{N}\).

d. The domain is given as \(\mathbb{Z}^+\). The range is \(\mathbb{Z}^+\) as well.

e. The domain is the set of bit strings. The range is the set of strings of 1’s, \(i.e., \{\lambda, 1, 11, 111, \ldots\}\), where \(\lambda\) is the empty string containing no symbols.

8. Rosen #1.8.8

The answers are: (a) 1, (b) 2, (c) -1, (d) 0, (e) 3, (f) -2, (g) \(\left\lceil \frac{1}{2} + 1 \right\rceil = \left\lceil \frac{3}{2} \right\rceil = 1\), (h) \(\left\lfloor 0 + 1 = \frac{1}{2} \right\rfloor = \left\lfloor \frac{3}{2} \right\rfloor = 2\)
9. Rosen #1.8.16
   a. \( f(n) = n + 17 \)
   b. \( f(n) = \lceil n/2 \rceil \)
   c. We set \( f(n) = n - 1 \) for even values of \( n \), and \( f(n) = n + 1 \) for odd values of \( n \).
      Thus, we have \( f(1) = 2 \), \( f(2) = 1 \), \( f(3) = 4 \), \( f(4) = 3 \), and so on. Note that his is just one function, even though its definition uses two formulae, depending on the parity of \( n \).
   d. \( f(n) = 17 \)

10. Rosen #1.8.52
From example 23 on p. 106, we know that one ATM cell is 53 bytes long, or \( 53 \times 8 = 424 \) bits long. Thus, in each case we need to divide the number of bits transmitted in 10 seconds by 424 and round down.
   a. In 10 seconds, this link can transmit \( 128,000 \cdot 10 = 1,280,000 \) bits. Therefore, the answer is \( \lceil 1,280,000/424 \rceil = 3018 \).
   b. In 10 seconds, this link can transmit \( 300,000 \cdot 10 = 3,000,000 \) bits. So, the answer is \( \lceil 3,000,000/424 \rceil = 7075 \).
   c. In 10 seconds, this link can transmit \( 1,000,000 \cdot 10 = 10,000,000 \) bits. So, the answer is \( \lceil 10,000,000/424 \rceil = 23,584 \).

11. Rosen #3.1.12
As one example, if \( a = 5 \) and \( b = 8 \), then the quadratic mean is \( ((5^2 + 8^2)/2)^{1/2} \approx 6.67 \), and the arithmetic mean is \( (5+8)/2 = 6.5 \). As another example, if \( a = 10 \) and \( b = 100 \), we find that the quadratic mean is approximately 71.06, and the arithmetic mean is 55. We conjecture that the quadratic mean of \( a \) and \( b \) is always greater than the arithmetic mean if \( a \) and \( b \) are distinct, positive, real numbers. To prove this result, we must show that \( ((a^2 + b^2)/2)^{1/2} > (a + b)/2 \). We prove it indirectly. We begin by squaring this relationship, which is an invertible operation because, by hypothesis, \( a \) and \( b \) are both positive. We then multiply by 4 to obtain \( 2a^2 + 2b^2 > a^2 + 2ab + b^2 \). Subtracting the right side from the left side, we obtain \( a^2 - 2ab + b^2 > 0 \), which is equivalent to \( (a - b)^2 > 0 \). Our result is proved.

12. Rosen #3.1.50
The decision problem has no input. The answer is always yes or always no, depending on whether the specific program with its specific program halts or not. In the former case, the decision procedure is “say yes,” and, in the latter case, it is “say no.”