

An Example of How to Compute a Log/AntiLog Table

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The polynomial

$$p(x) = x^4 + x + 1$$

is primitive (hence, irreducible) over $GF(2)$. We now use $p(x)$ to construct a log/antilog table for $GF(2^4)$. Let $\xi = x \bmod p(x)$.

Log	AntiLog <small>$a_3a_2a_1a_0$</small>	Calculation
$-\infty$	0000	$\xi^{-\infty} = 0$
0	0001	$\xi^0 = 1$
1	0010	$\xi^1 = \xi$
2	0100	$\xi^2 = \xi^2$
3	1000	$\xi^3 = \xi^3$
4	0011	$\xi^4 = \xi + 1$
5	0110	$\xi^5 = \xi^2 + \xi$
6	1100	$\xi^6 = \xi^3 + \xi^2$
7	1011	$\xi^7 = \xi^4 + \xi^3 = (\xi + 1) + \xi^3 = \xi^3 + \xi + 1$
8	0101	$\xi^8 = \xi^4 + \xi^2 + \xi = (\xi + 1) + \xi^2 + \xi = \xi^2 + 1$
9	1010	$\xi^9 = \xi^3 + \xi$
10	0111	$\xi^{10} = \xi^4 + \xi^2 = (\xi + 1) + \xi^2 = \xi^2 + \xi + 1$
11	1110	$\xi^{11} = \xi^3 + \xi^2 + \xi$
12	1111	$\xi^{12} = \xi^4 + \xi^3 + \xi^2 = (\xi + 1) + \xi^3 + \xi^2 = \xi^3 + \xi^2 + \xi + 1$
13	1101	$\xi^{13} = \xi^4 + \xi^3 + \xi^2 + \xi = (\xi + 1) + \xi^3 + \xi^2 + \xi = \xi^3 + \xi^2 + 1$
14	1001	$\xi^{14} = \xi^4 + \xi^3 + \xi = (\xi + 1) + \xi^3 + \xi = \xi^3 + 1$

Check: One can check this answer by verifying that $\xi^{15} = 1$, i.e.,

$$\xi^{15} = \xi(\xi^3 + 1) = \xi^4 + \xi = (\xi + 1) + \xi = 1$$