Homework 3

- **Listening Assignment**: Listen to Rachmanioff’s Piano Concerto No. 4
- **Read** [http://www.csee.umbc.edu/~lomonaco/f06/653/handouts/Peterson-Pages22-25.pdf](http://www.csee.umbc.edu/~lomonaco/f06/653/handouts/Peterson-Pages22-25.pdf)

1UG) Consider the following degree 4 irreducible polynomial $p(x)$ given in Peterson's Table of Irreducible Polynomials over $\mathbb{GF}(2)$

| DEGREE | 4 ... | 3 37D ... |

a) Write down $p(x)$.

b) Since $p(x)$ is irreducible and of degree 3, it follows that

$$ \mathbb{GF}(2^4) = \mathbb{GF}(2)[x] \mod p(x) $$

List all the elements of $\mathbb{GF}(2^4)$ in the above representation, i.e., in terms of $\xi = x \mod p(x)$

c) Let $\xi = x \mod p(x)$. Why is $\{ \xi^k \}$ not a complete list of all the non-zero elements of $\mathbb{GF}(2^4)$?

2UG) Consider the following matrix over $\mathbb{GF}(2)$

$$ M = \begin{pmatrix}
0 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 1
\end{pmatrix} $$

a) Prove that the rows of $M$ are linearly dependent.

b) Prove that the first three rows $M$ form a basis for the row space of $M$.

c) What is the dimension of the row space of $M$? Explain your answer.
3UG) Consider the following matrix $S$ over $\text{GF}(3)$

$$
S = \begin{pmatrix}
0 & 0 & 2 & 2 & 0 & 2 \\
2 & 2 & 0 & 2 & 1 & 2 \\
1 & 1 & 2 & 0 & 2 & 2 \\
1 & 1 & 0 & 1 & 2 & 1
\end{pmatrix}
$$

a) Put the matrix $S$ into echelon canonical form. \textbf{(Hint. See section 2.6 of optional text)}

b) Use the resulting echelon canonical form to find a basis for the row space of $S$. Explain your answer.

c) What is the dimension of the row space of $S$? Explain how you found your answer.