1) Consider the following degree 4 irreducible polynomial $p(x)$ given in Peterson's Table of Irreducible Polynomials over $GF(2)$

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a) Write down $p(x)$.

b) Since $p(x)$ is irreducible and of degree 4, it follows that

$$GF(2^4) = GF(2)[x] \mod p(x)$$

List all the elements of $GF(2^4)$ in the above representation, i.e., in terms of $\xi = x \mod p(x)$

c) Let $\xi = x \mod p(x)$. Why is $\{\xi^k\}$ not a complete list of all the non-zero elements of $GF(2^4)$?

2) Let $V$ be the cyclic code in $R_{15} = GF(2)[x]/(x^{15} + 1)$ given by the generator polynomial $g(x) = x^8 + x^4 + x^2 + x + 1$.

a) What is the length $n$ of $V$?

b) What is the dimension $k$ of $V$?

c) Use the generator polynomial $g(x)$ to construct a generator matrix $G$ for $V$.

d) What is the parity check polynomial $h(x)$ of $V$?

e) Use the parity check polynomial $h(x)$ to construct the parity check matrix $H$ of $V$. 
3) Given that

\[ x^9 + 1 = (x + 1)(x^2 + x + 1)(x^6 + x^3 + 1) \]

is a complete factorization over \( GF(2) \) of \( x^9 + 1 \) into irreducible polynomials,

a) Draw the lattice of all ideals in \( R_9 = GF(2)[x]/(x^9 + 1) \).

b) Determine the dimension of each ideal in \( R_9 \).

c) Determine the number of elements in each ideal in \( R_9 \).

d) List all the elements of the ideals

\( (x^6 + x^3 + 1) \) and \( (x+1)(x^6 + x^3 + 1) \)