CMSC 442/653  
Instructor: Dr. Lomonaco

Homework 1

- **Listening Assignment:** Listen to Beethoven’s 5-th symphony.


1) Construct the multiplication table of the group of symmetries of the equilateral triangle given by the presentation 
\[(\rho, \sigma : \rho^3 = 1, \sigma^2 = 1, \rho \sigma = \sigma \rho^2)\]
Assume that the distinct group elements are:
\[1, \rho, \rho^2, \sigma, \rho \sigma, \rho^2 \sigma\]

2) Construct the multiplication table of the group of symmetries of the square given by the presentation 
\[(\rho, \sigma : \rho^4 = 1, \sigma^2 = 1, \rho \sigma = \sigma \rho^3)\]
Assume that the distinct group elements are:
\[\{\rho^m \sigma^n : 0 \leq m < 4, 0 \leq n < 2\}\]

**Additional problem for grad students in CMSC 653:**

**Grad3)** Let \(S\) be a set with an associative binary operation \(\cdot: S \times S \to S\). Let \(e_L\) be a left identity of \(S\) (i.e., \(e_L \cdot s = s \forall s \in S\)), and let \(e_R\) be a right identity of \(S\) (i.e., \(s \cdot e_R = s \forall s \in S\)).

a) Prove that \(e_L = e_R\).

b) Also prove that \(S\) can have at most one 2-sided identity.

**Grad4)** Let \(S\) be a set with an associative binary operation \(\cdot: S \times S \to S\) and a 2-sided identity \(e\), and let \(s \in S\). Let \(s_L\) and \(s_R\) be elements of \(S\) such that 
\[s_L \cdot s = e = s \cdot s_R\]
Prove that \(s_L = s_R\).