Quantum Computing

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Overview
Four Talks

Elementary

- ✓ A Rosetta Stone for Quantum Computation
- ✓ Three Quantum Algorithms
- Quantum Hidden Subgroup Algorithms
- An Entangled Tale of Quantum Entanglement

Advanced

Lecture 3

Continuous Quantum Hidden Subgroup Algorithms

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Louis H. Kauffman

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Existing Quantum Algorithms

- Hidden Subgroup Algorithms – Shor-Like Algorithms
- Amplitude amplification – Grover-Like Algorithms
- Quantum Algorithms Simulating Quantum Systems
- Sipser's Algorithm
- Adiabatic Algorithms ???

Some Existing HSA's

- Hidden subgroup algorithms
  - Deutsch-Jozsa
  - Simon
  - Shor
  - Legendre symbol
  - Hallgen
  - Various Non-abel. Algorithms
  - Others

We will now discuss the following Six HSA's

- Continuous Shor on $\mathbb{R}$
- Wandering Shor
- Lift of Shor to $\mathbb{Z}$
- HSA on Circle
- Dual Shor HSA
- HSA for Functional Integrals

These Six Algorithms Can Be Found in the Following Three Papers

- Lomonaco & Kauffman, Quantum Hidden Subgroup Algorithms: *A Mathematical Perspective*, AMS, CONM/305, (2002), 139-202. http://xxx.lanl.gov/abs/quant-ph/0201095
- Lomonaco & Kauffman, A Continuous Variable Shor Algorithm, http://xxx.lanl.gov/abs/quant-ph/0210141
- Lomonaco & Kauffman, Continuous Quantum Hidden Subgroup Algorithms, http://xxx.lanl.gov/abs/quant-ph/0304084

Hidden Subgroup Algorithms

Hidden Subgroup Structure

Ambient Group — Target Set

Def. A Map $\varphi : A \to S$ is said to have hidden subgroup structure if there exist

Hidden Subgroup

- A subgroup K_φ of A, and
- An injection $\iota_\varphi : A / K_\varphi \to S$

Set of Right Cosets

s. t. the diagram

$$\begin{array}{ccc} A & \xrightarrow{\varphi} & S \\ & \nu \searrow & \uparrow \iota_\varphi \\ & & A / K_\varphi \end{array}$$

Hidden Natural Surjection

is commutative.

Hidden Subgroup Structure (Cont.)

Hidden Epimorphism

Hidden Quotient Group

$$\begin{array}{ccc} A & \xrightarrow{\varphi} & S \\ & \nu \searrow & \uparrow \iota_\varphi \\ & & A/K_\varphi \end{array}$$

If K_φ is an invariant subgroup of A, then

$$H_\varphi = A/K_\varphi$$

is a group, and $\nu : A \to A/K_\varphi$ is an epimorphism

Origin of QHS Algorithms

Shor's Quantum factoring algorithm reduces the task of factoring an integer N to the task of finding the period P of a function

$$\begin{array}{ccc} \mathbb{Z} & \xrightarrow{\varphi} & \mathbb{Z} \bmod N \\ n & \mapsto & a^n \bmod N \end{array}$$

Kitaev observed that finding the period P is equivalent to finding the subgroup $P\mathbb{Z} \subset \mathbb{Z}$, i.e., the kernel of φ.

Quantum Hidden Subgroup Algorithms

Quantum Algorithm	Ambient Gp A	Hidden Subgp K_φ
Deutsch-Jozsa	$\mathbb{Z}_2$	$K_\varphi = \begin{cases} \{0\} \\ \mathbb{Z}_2 \end{cases}$
Simon	$\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \cdots \oplus \mathbb{Z}_2$	$K_\varphi \cong \mathbb{Z}_2$
Shor Factoring	$\mathbb{Z}$	$K_\varphi = P\mathbb{Z}$

The Hidden Subgroup Problem (HSP)

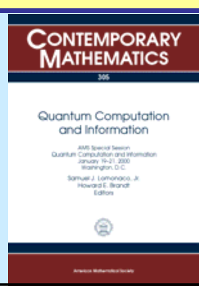
Given a map

$$\varphi : A \to S$$

with hidden subgroup structure, determine the hidden subgroup K_φ of the ambient group A. An algorithm solving this problem is called a hidden subgroup algorithm (HSA)

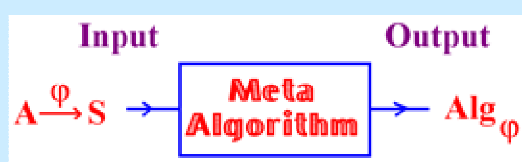
The First of the Three Papers

- Lomonaco & Kauffman, Quantum Hidden Subgroup Algorithms: *A Mathematical Perspective*, AMS, CONM/305, (2002). http://xxx.lanl.gov/abs/quant-ph/0201095



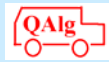
The Quantum Hidden Subgroup Paper Shows how to create a

Meta Algorithm

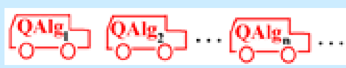


An Analogy

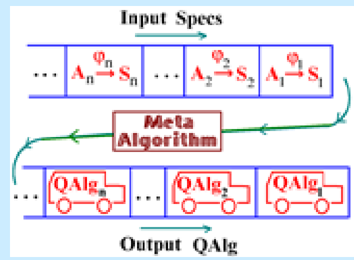
- Autos before Henry Ford



- Autos after Henry Ford



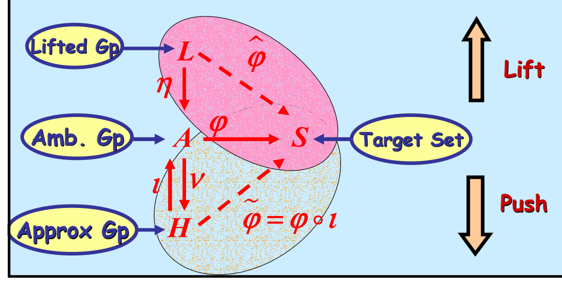
Quantum Version of Henry Ford's Assembly Line



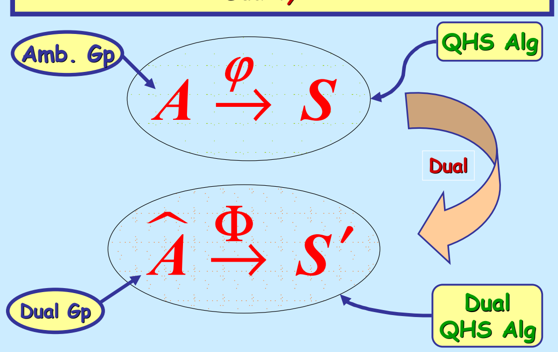
Three Methods for Creating New Quantum Algorithms

Two Ways to Create New Quantum Algorithms
Lifting and Pushing

Given $\varphi : A \to S$



A 3rd Way to Create New Quantum Algorithms
Duality



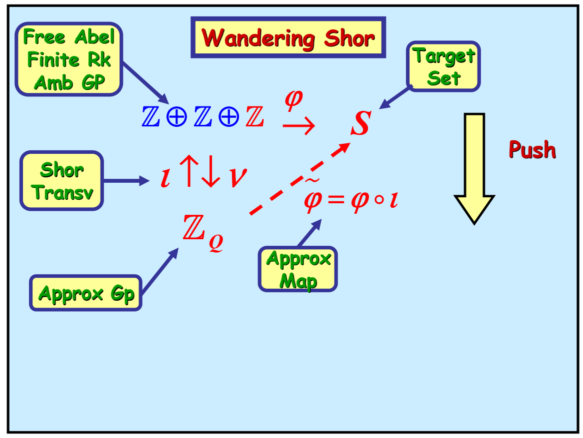
Summary
3 Ways to create New Quantum Algorithms

- Lifting
- Pushing
- Duality

Hidden Subgroup Algorithms

Some Past Algorithms

- **Wandering Shor**
  - Lomonaco & Kauffman, Quantum Hidden Subgroup Algorithms: *A Mathematical Perspective*, AMS, CONM/305, (2002). http://xxx.lanl.gov/abs/quant-ph/0201095
- **Continuous Shor**
  - Lomonaco & Kauffman, A Continuous Variable Shor Algorithm, http://xxx.lanl.gov/abs/quant-ph/0210141

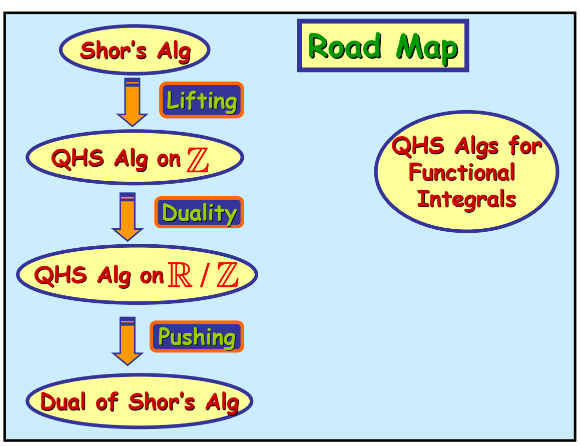


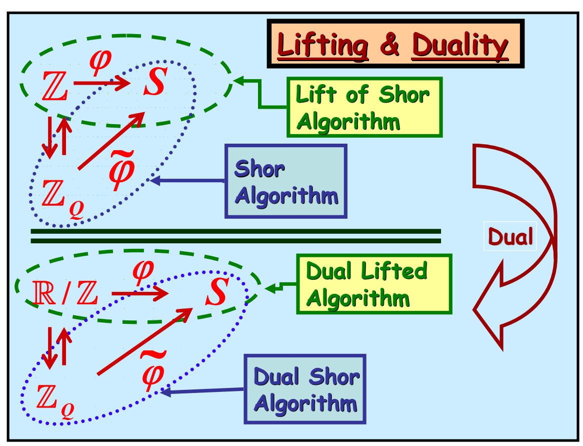
Continuous Shor

Key Idea: Lifting of discrete algorithms to a continuous groups

Three Recent QHS Algorithms

- A quantum algorithm on the Circle
- A quantum algorithm dual to Shor's algorithm
- A highly speculative quantum algorithm for functional integrals
  $\Longrightarrow$? Quantum algorithm for the Jones polynomial





A Lifting of Shor's Quantum Factoring Algorithm to Integers $\mathbb{Z}$

A Momentary Digression

Fourier Analysis on the Circle

The Circle as a Group

The circle group can be viewed as

- A multiplicative group, i.e., as the unit circle in the complex plane $\mathbb{C}$

$$\left\{ e^{2\pi i x} : x \in \mathbb{R} \right\}$$

$$e^{2\pi i x} \bullet e^{2\pi i y} = e^{2\pi i (x+y)}$$

where $\mathbb{R}$ denotes the additive group of reals.

The Circle as a Group

The circle group can *also* be viewed as

- An additive group, i.e., as

$$\mathbb{R}/\mathbb{Z} = reals \bmod 1$$

$$x + y \bmod 1$$

where $\mathbb{Z}$ denotes the additive group of integers.

The Character Group

The character group $\widehat{A}$ of an abelian group A is defined as

$$\widehat{A} = Hom(A, Circle)$$

$$= \{ \chi : A \to Circle : \chi \text{ a morphism} \}$$

with group operation (in multiplicative notation),

$$(\chi_1 \bullet \chi_2)(a) = \chi_1(a) \bullet \chi_2(a)$$

or (in additive notation) as

$$(\chi_1 + \chi_2)(a) = \chi_1(a) + \chi_2(a)$$

The Character Groups of $\mathbb{Z}$ and $\mathbb{R}/\mathbb{Z}$

- The character group of $\mathbb{Z}$ is

$$\widehat{\mathbb{Z}} = \left\{ \chi_x : n \mapsto e^{2\pi i n x} : x \in \mathbb{R} \right\} = \mathbb{R}/\mathbb{Z}$$

- The character group of $\mathbb{R}/\mathbb{Z}$ is

$$\widehat{\mathbb{R}/\mathbb{Z}} \cong \left\{ \chi_n : x \mapsto e^{2\pi i n x} : n \in \mathbb{Z} \right\}$$

$$\cong \left\{ \chi_n : x \mapsto nx \bmod 1 : n \in \mathbb{Z} \right\} = \mathbb{Z}$$

Discrete

$$\mathbb{Z} \Leftrightarrow \mathbb{R}/\mathbb{Z}$$

Continuous

Fourier Analysis on the Circle $\mathbb{R}/\mathbb{Z}$

The *Fourier transform* of $f:\mathbb{R}/\mathbb{Z}\to\mathbb{C}$ is defined as the map

$$\widehat{f}:\mathbb{Z}\to\mathbb{C}$$

given by

$$\widehat{f}(n)=\oint dx e^{-2\pi i n x} f(x)$$

The *inverse Fourier transform* is defined as

$$f(x)=\sum_{n\in\mathbb{Z}} e^{2\pi i n x}\widehat{f}(n)$$

Needed Mathematical Machinery

- Dirac Delta function $\delta(x)$ on $\mathbb{R}/\mathbb{Z}$

- For P a non-zero integer, we will also need on $\mathbb{R}/\mathbb{Z}$ the generalized function

$$\delta_P(x)=\frac{1}{|P|}\sum_{n=0}^{P-1}\delta\left(x-\frac{n}{P}\right)$$

Rigged Hilbert Space

- $\mathcal{H}_{\mathbb{R}/\mathbb{Z}}$ denotes the rigged Hilbert space on $\mathbb{R}/\mathbb{Z}$ with orthonormal basis

$\{|x\rangle : x\in\mathbb{R}/\mathbb{Z}\}$, i.e., $\langle x|y\rangle=\delta(x-y)$

- The elements of $\mathcal{H}_{\mathbb{R}/\mathbb{Z}}$ are formal integrals of the form

$$\oint dx\, f(x)|x\rangle$$

Finally, let $\mathcal{H}_{\mathbb{Z}}$ denote the space of formal sums

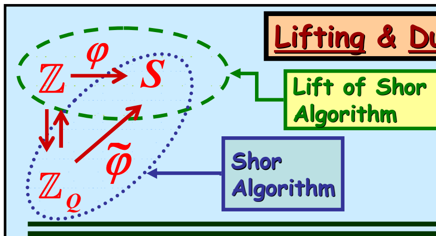
$$\left\{\sum_{n=-\infty}^{\infty} a_n|n\rangle : a_n\in\mathbb{C}\quad \forall n\in\mathbb{Z}\right\}$$

with orthonormal basis

$$\{|n\rangle : n\in\mathbb{Z}\}$$

A Lifting of Shor's Quantum Factoring Algorithm to Integers $\mathbb{Z}$

Lifting & Duality



Periodic Functions on $\mathbb{Z}$

Let $\varphi : \mathbb{Z} \to \mathbb{C}$ be periodic function with hidden minimum period P .

OBJECTIVE:

Find P

- Step 0. Initialize

$$|\psi_0\rangle = |0\rangle|0\rangle \in \mathcal{H}_{\mathbb{R}/\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

- Step 1. Apply $\mathcal{F}^{-1} \otimes 1$

$$|\psi_1\rangle = \sum_{n\in\mathbb{Z}} e^{2\pi i n \cdot 0}|n\rangle|0\rangle = \sum_{n\in\mathbb{Z}}|n\rangle|0\rangle \in \mathcal{H}_{\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

- Step 2. Apply $U_\varphi : |n\rangle|u\rangle \mapsto |n\rangle|u+\varphi(n)\rangle$

$$|\psi_2\rangle = \sum_{n\in\mathbb{Z}} |n\rangle|\varphi(n)\rangle$$

- Step 3. Apply $\mathcal{F} \otimes 1$

$$\begin{aligned}
|\psi_3\rangle &= \oint dx\,|x\rangle \sum_{n\in\mathbb{Z}} e^{-2\pi i n x}|\varphi(n)\rangle \in \mathcal{H}_{\mathbb{R}/\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}} \\
&= \oint dx\,|x\rangle \sum_{n_1\in\mathbb{Z}} \sum_{n_0=0}^{P-1} e^{-2\pi i (n_1 P + n_0)x}|\varphi(n_1 P + n_0)\rangle \\
&= \oint dx\,|x\rangle \left(\sum_{n_1\in\mathbb{Z}} e^{-2\pi i n_1 P x}\right) \sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x}|\varphi(n_0)\rangle \\
&= \oint dx\,|x\rangle \delta_P(x) \sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x}|\varphi(n_0)\rangle \\
&= \sum_{n=0}^{P-1}\left|\frac{n}{P}\right\rangle \left(\frac{1}{P}\sum_{n_0=0}^{P-1} e^{-2\pi i n_0 x}|\varphi(n_0)\rangle\right) \\
&= \sum_{n=0}^{P-1}\left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle
\end{aligned}$$

- Step 4. Measure

$$|\psi_3\rangle = \sum_{n=0}^{P-1}\left|\frac{n}{P}\right\rangle \left|\Omega\left(\frac{n}{P}\right)\right\rangle$$

with respect to the observable

$$\mathcal{O} = \oint dy \frac{\lfloor Qy \rfloor}{Q}|y\rangle\langle y|$$

to produce a random eigenvalue m/Q and then proceed to find the corresponding n/P using the continued fraction recursion. (We assume $Q \geq 2P^2$)

The Actual Un-Lifted Shor Algorithm

The Actual (Un-Lifted) Shor Algorithm

Make the following approximations by selecting a sufficiently large integer Q :

$$\mathbb{Z} \approx \mathbb{Z}_Q = \{k \in \mathbb{Z} : 0 \leq k < Q\}$$

$$\mathbb{R}/\mathbb{Z} \approx \mathbb{Z}_Q = \left\{\frac{r}{Q} \bmod 1 : r = 0,1,\ldots,Q-1\right\}$$

$$\varphi : \mathbb{Z} \to \mathbb{C} \quad \approx \tilde{\varphi} : \mathbb{Z}_Q \to \mathbb{C}$$

$\tilde{\varphi}$ is only approximately periodic !

Run the algorithm in

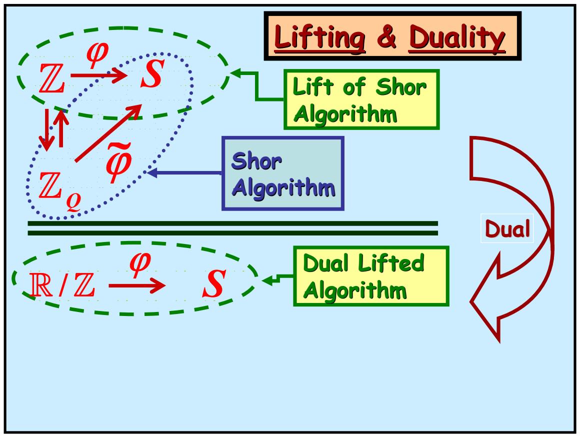
$$\mathcal{H}_{\mathbb{Z}_Q} \otimes \mathcal{H}_S$$

and measure the observable

$$\mathcal{O} = \sum_{r=0}^{Q-1} \frac{r}{Q} \left| \frac{r}{Q} \right\rangle \left\langle \frac{r}{Q} \right|$$

A Quantum Hidden Subgroup Algorithm on the *Circle*

The Dual Algorithm on the Circle



Rigged Hilbert Space

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$$\{|x\rangle : x \in \mathbb{R}/\mathbb{Z}\}$$, i.e., $$\langle x | y \rangle = \delta(x-y)$$

- The elements of $\mathcal{H}_{\mathbb{R}/\mathbb{Z}}$ are formal integrals of the form

$$\oint dx\, f(x) |x\rangle$$

Finally, let $\mathcal{H}_{\mathbb{Z}}$ denote the space of formal sums

$$\left\{ \sum_{n=-\infty}^{\infty} a_n |n\rangle : a_n \in \mathbb{C} \quad \forall n \in \mathbb{Z} \right\}$$

with orthonormal basis

$$\{|n\rangle : n \in \mathbb{Z}\}$$

Periodic Admissible Functions on $\mathbb{R}/\mathbb{Z}$

Let $f:\mathbb{R}/\mathbb{Z}\to\mathbb{C}$ be an admissible periodic function of minimum rational period $\alpha\in\mathbb{Q}/\mathbb{Z}$

Proposition:
Let $\alpha = a_1/a_2$ (with $\gcd(a_1,a_2)=1$) be a period of f . Then $1/a_2$ is also a period of f .

Remark: Hence, the minimum rational period is always the reciprocal of an integer modulo 1 .

- **Step 0.** Initialize

$$|\psi_0\rangle = |0\rangle|0\rangle \in \mathcal{H}_{\mathbb{Z}}\otimes\mathcal{H}_{\mathbb{C}}$$

- **Step 1.** Apply $\mathcal{F}^{-1}\otimes 1$

$$|\psi_1\rangle = \oint dx e^{2\pi i x\cdot 0}|x\rangle|0\rangle = \oint dx|x\rangle|0\rangle \in \mathcal{H}_{\mathbb{R}/\mathbb{Z}}\otimes\mathcal{H}_{\mathbb{C}}$$

- **Step 2.** Apply $U_\varphi : |x\rangle|u\rangle \mapsto |x\rangle|u+\varphi(x)\rangle$

$$|\psi_2\rangle = \oint dx|x\rangle|\varphi(x)\rangle$$

- **Step 3.** Apply $\mathcal{F}\otimes 1$

$$\begin{aligned}|\psi_3\rangle &= \sum_{n\in\mathbb{Z}}\oint dx\, e^{-2\pi i n x}|n\rangle|\varphi(x)\rangle \\ &= \sum_{n\in\mathbb{Z}}|n\rangle\oint dx\, e^{-2\pi i n x}|\varphi(x)\rangle \in \mathcal{H}_{\mathbb{Z}}\otimes\mathcal{H}_{\mathbb{C}}\end{aligned}$$

Letting $x_m = x - \frac{m}{a}$, we have

$$\begin{aligned}\oint dx\, e^{-2\pi i n x}|\varphi(x)\rangle &= \sum_{m=0}^{a-1}\int_{\frac{m}{a}}^{\frac{m+1}{a}} dx\, e^{-2\pi i n x}|\varphi(x)\rangle \\ &= \sum_{m=0}^{a-1}\int_0^{\frac{1}{a}} dx_m\, e^{-2\pi i n\left(x_m+\frac{m}{a}\right)}\left|\varphi\left(x_m+\frac{m}{a}\right)\right\rangle \\ &= \left(\sum_{m=0}^{a-1} e^{-\frac{2\pi i n m}{a}}\right)\int_0^{\frac{1}{a}} dx\, e^{-2\pi i n x}|\varphi(x)\rangle\end{aligned}$$

But $\sum_{m=0}^{a-1} e^{-\frac{2\pi i n m}{a}} = a\delta_{n=0 \bmod a} = \begin{cases} a & \text{if } n = 0 \bmod a \\ 0 & \text{otherwise}\end{cases}$

Thus,

$$\begin{aligned}|\psi_3\rangle &= \sum_{n\in\mathbb{Z}}|n\rangle\oint dx\, e^{-2\pi i n x}|\varphi(x)\rangle \\ &= \sum_{n\in\mathbb{Z}}|n\rangle\delta_{n=0 \bmod a}\int_0^{1/a} dx\, e^{-2\pi i n x}|\varphi(x)\rangle \\ &= \sum_{\ell\in\mathbb{Z}}|\ell a\rangle\left(\int_0^{1/a} dx\, e^{-2\pi i \ell a x}|\varphi(x)\rangle\right) \\ &= \sum_{\ell\in\mathbb{Z}}|\ell a\rangle|\Omega(\ell a)\rangle\end{aligned}$$

- **Step 4.** Measure

$$|\psi_3\rangle = \sum_{\ell\in\mathbb{Z}}|\ell a\rangle|\Omega(\ell a)\rangle$$

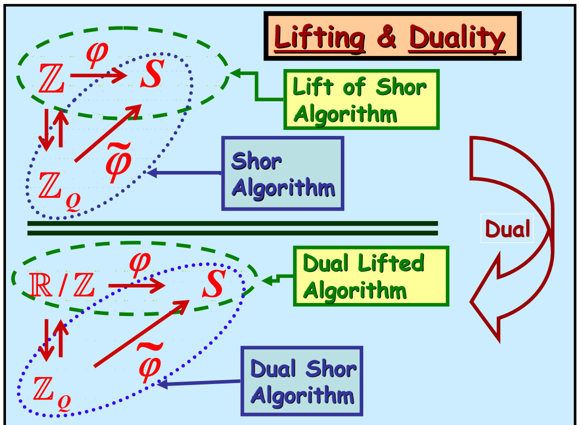
with respect to the observable

$$\mathcal{O} = \sum_{n\in\mathbb{Z}} n|n\rangle\langle n|$$

to produce a random eigenvalue ℓa

The corresponding discrete algorithm

The Algorithmic Dual of Shor's Quantum Factoring Algorithm



We now create a corresponding discrete algorithm

The approximations are:

$$\mathbb{Z} \approx \mathbb{Z}_Q = \{k \in \mathbb{Z} : 0 \le k < P\}$$

$$\mathbb{R}/\mathbb{Z} \approx \mathbb{Z}_Q = \left\{ \frac{r}{Q} \bmod 1 : r = 0, 1, \ldots, Q-1 \right\}$$

$$\varphi : \mathbb{Z} \to \mathbb{C} \quad \approx \tilde{\varphi} : \mathbb{Z}_Q \to \mathbb{C}$$

$\tilde{\varphi}$ is only approximately periodic !

Run the algorithm in

$$\mathcal{H}_{\mathbb{Z}_Q} \otimes \mathcal{H}_S$$

and measure the observable

$$\mathcal{O} = \sum_{k=0}^{Q-1} k \, |k\rangle\langle k|$$

Quantum Algorithms based on Feynman Functional integrals

Caveat Emptor

The following algorithm is highly speculative.

In the spirit of Feynman, the following quantum algorithm is based on functional integrals whose existence is difficult to determine, let alone approximate.

The Space Paths

Paths = all continuous paths $x:[0,1]\to\mathbb{R}^n$ which are L^2 with respect to the inner product

$$x\bullet y=\int_0^1 ds\, x(s)\bullet y(s)$$

Paths is a vector space over $\mathbb{R}$ with respect to

$$\begin{cases}(\lambda x)(s) & = & \lambda x(s)\\ (x+y)(s) & = & x(s)+y(s)\end{cases}$$

The Problem to be Solved

Let $\varphi: Paths\to\mathbb{C}$ be a functional with a hidden subspace V of $Paths$ such that

$$\varphi(x+v)=\varphi(x)\quad \forall v\in V$$

Objective. Create a quantum algorithm that finds the hidden subspace V .

The Ambient Rigged Hilbert Space

Let $\mathcal{H}_{Paths}$ be the rigged Hilbert space with orthonormal basis ,

$$\{|x\rangle : x\in Paths\}$$

and with bracket product

$$\langle x\,|\,y\rangle=\delta(x-y)$$

Parenthetical Remark

Please note that $Paths$ can be written as the following disjoint union:

$$Paths=\bigcup_{v\in V}\left(v+V^{\perp}\right)$$

- Step 0. **Initialize** $|\psi_0\rangle=|0\rangle|0\rangle\in\mathcal{H}_{Paths}\otimes\mathcal{H}_{\mathbb{C}}$
- Step 1. **Apply** $\mathcal{F}^{-1}\otimes 1$

$$|\psi_1\rangle=\int_{Paths}\mathcal{D}x\;\; e^{2\pi i x\bullet 0}|x\rangle|0\rangle=\int_{Paths}\mathcal{D}x\,|x\rangle|0\rangle$$

- Step 2. **Apply** $U_\varphi:|x\rangle|u\rangle\mapsto|x\rangle|u+\varphi(x)\rangle$

$$|\psi_2\rangle=\int_{Paths}\mathcal{D}x\,|x\rangle|\varphi(x)\rangle$$

- Step 3. **Apply** $\mathcal{F}\otimes 1$

$$\begin{aligned}|\psi_3\rangle &= \int_{Paths}\mathcal{D}y\int_{Paths}\mathcal{D}x\, e^{-2\pi i x\bullet y}|y\rangle|\varphi(x)\rangle\\ &= \int_{Paths}\mathcal{D}y\,|y\rangle\int_{Paths}\mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle\end{aligned}$$

But

$$\int_{Paths} \mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle = \int_V \mathcal{D}v \int_{v+V^\perp} \mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle$$
$$= \int_V \mathcal{D}v \int_{V^\perp}\mathcal{D}x\, e^{-2\pi i(v+x)\bullet y}|\varphi(v+x)\rangle$$
$$= \int_V \mathcal{D}v\, e^{-2\pi i v\bullet y}\int_{V^\perp}\mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle$$

However, $\int_V \mathcal{D}v\, e^{-2\pi i v\bullet y} = \int_{V^\perp}\mathcal{D}u\,\delta(y-u)$

So,

$$|\psi_3\rangle = \int_{Paths_n} \mathcal{D}y\,|y\rangle \int_V \mathcal{D}v\, e^{-2\pi i v\bullet y} \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle$$
$$= \int_{Paths_n} \mathcal{D}y\,|y\rangle \int_{V^\perp} \mathcal{D}u\,\delta(y-u) \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\bullet y}|\varphi(x)\rangle$$
$$= \int_{V^\perp} \mathcal{D}u\,|u\rangle \int_{V^\perp} \mathcal{D}x\, e^{-2\pi i x\bullet u}|\varphi(x)\rangle$$
$$= \int_{V^\perp} \mathcal{D}u\,|u\rangle|\Omega(u)\rangle$$

- Step 4. **Measure**

$$|\psi_3\rangle = \int_{V^\perp} \mathcal{D}u\,|u\rangle|\Omega(u)\rangle$$

with respect to the observable

$$A = \int_{Paths} \mathcal{D}w\, w\,|w\rangle\langle w|$$

to produce a random element of $V^\perp$

Question

Can the above path integral quantum algorithm be modified in such a way as to create a quantum algorithm for the Jones polynomial ?

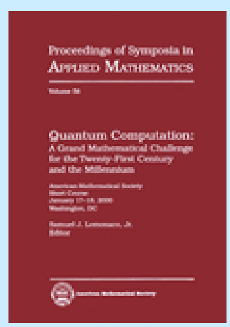
I.e., can it be modified by replacing *Paths* by the **space of gauge connections**, and by making suitable modifications?

$$\widehat{\psi}(K) = \int \mathcal{D}A\, \psi(A)\, \mathcal{W}_K(A)$$

where $\mathcal{W}_K(A)$ is the **Wilson loop**

$$\mathcal{W}_K(A) = tr\left(P\exp\left(\oint_K A\right)\right)$$

Quantum Computation: *A Grand Mathematical Challenge for the Twenty-First Century and the Millennium*, **Samuel J. Lomonaco, Jr. (editor)**, AMS PSAPM/58, (2002).



Quantum Computation and Information, **Samuel J. Lomonaco, Jr. and Howard E. Brandt (editors)**, AMS CONM/305, (2002).