Exercise 27.2-2 (New Version)
Proof:
I: A comparison network with \( n \) inputs correctly sorts the \( n-1 \) sequences \(<1,0,\ldots,0>, <1,1,0,\ldots,0>, \ldots, <1,1,1,\ldots,1,0>\) → it sorts \( n \) inputs sequence \(<n,n-1,\ldots,1>\) correctly.

Suppose for the purpose of contradiction that the network can correctly sort \( n-1 \) zero-one sequences, but not the \( n \) input sequences \(<n,n-1, n-2, \ldots, 1>\).
Then for input \(<n, n-1, j,i \ldots, 1>\) (\( j>i \)), there exists an output sequence that can be \(<n, n-1, i, j, \ldots, 1>\) when \( i<j \). Define a monotonically increasing function \( f \) such that \( f(x)=0 \) if \( x<j \), \( f(x)=1 \) if \( x \geq j \). Apply Lemma 27.1, the \(<n,n-1,j,i, \ldots, 1>\) input will be \(<1,1,1,0,\ldots,0>\), and the \(<n,n-1, i, j, \ldots, 1>\) output will be \(<1,1,0,1,\ldots,0>\). We obtain the contradiction that the network fails to sort the zero-one sequence \(<1,1,1,0,\ldots,0>\).

II: A comparison network with \( n \) inputs sorts \( n \) inputs sequence \(<n,n-1,\ldots,1>\) correctly \(→\) it correctly sorts the \( n-1 \) sequences \(<1,0,\ldots,0>, <1,1,0,\ldots,0>, \ldots, <1,1,1,\ldots,1,0>\).
Suppose for the purpose of contradiction that the network can correctly sort \( n \) inputs sequence \(<n,n-1,j,i, \ldots, 1>\) (\( j>i \)), but not \( n-1 \) sequences \(<1,0,\ldots,0>, <1,1,0,\ldots,0>, \ldots, <1,1,1,\ldots,1,0>\).
We can define function \( f \) such that \( f(x)=1 \) if \( x \geq j \), \( f(x)=0 \) if \( x<j \). Apply Lemma 27.1, so the network can correctly sort \(<1,1,\ldots,1,0,\ldots,0>\) that is a contradiction to our assumption.

So the statement of 27.2-2 is TRUE.

Exercise 27.2-5 (New Version)
Proof: Suppose for the purpose of contradiction that the statement of 27.2-5 is NOT true. Suppose the input is drawn from set \{1,0\}, for an example:

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0
i
j
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If there is no comparator between 1 and 0, 0 can never be placed before 1 in the output, so the sorting network can’t produce a correct answer.
Because of the Fact that for every input sequence, sorting network should produce a monotonically increasing output sequence. We can always find an example such that input \( i \) is 1, input \( j \) is 0, \( j>i \), input \(<i \) are all 0s, and input \( >j \) are all 1s. So, if there is no comparator between \( i \) and \( j \), 0 can never be placed before 1 in the output. That is a contradiction to the property of sorting network. So the statement of 27.2-5 is TRUE.

Exercise 27.3-6 (New Version)
Fact: By Lemma 27.3, the HALF CLEANER produces two bitonic sequences of half the size such that every element in the top half is at least as small as every element in the bottom half.
**Proof:** Suppose for the purpose of contradiction that the network sorts all zero-one bitonic sequences but there exists a sequence of arbitrary numbers that the network doesn’t sort correctly. That is, there exists an input bitonic sequence $<a_1, a_2, \ldots, a_n>$ containing elements $a_i$ and $a_j$ such that $a_i < a_j$, but the network places $a_j$ before $a_i$ in the output sequence.

We define a monotonically increasing function $f$: $f(x)=0$ if $x \leq a_i$, $f(x)=1$ if $x > a_i$.

By lemma 27.1, the output of the network places $f(a_j)$ before $f(a_i)$, that is 1 is placed before 0. That is a contradiction. Any bitonic sequence can be recursively reduced as the BITONIC-SORTER does, and can be replaced by 0 and 1 using the function defined above. So such proof above can be used to prove any input conditions.