CMSC 641
HW1: Review Exercises
Due Date: 7th February

January 30, 2002

1 Probability Distribution [5+7+6]

Proposition:- Let \( m \geq 0 \) and \( 0 < \epsilon < 1 \). Let \( X_1, X_2, X_3, \ldots \) and \( S_0, S_1, S_2, \ldots \) be non-negative integer valued random variables, such that

\[
S_n = \sum_{i=1}^{n} X_i \leq m
\]

\[
E(X_{n+1}|S_n) \geq \epsilon(m - S_n)
\]

Show that

1. \( E(S_n) \geq m(1 - (1 - \epsilon)^n) \)

2. Using definition of expectation also show that \( E(S_n) \leq m - 1 + \Pr(S_n = m) \) and therefore \( \Pr(S_n = m) \geq 1 - m(1 - \epsilon)^n \)

3. Conclude that the expected least \( n \) such that \( S_n = m \) is \( O(\log m) \)

2 Recurrence Relation [3+4+5]

Solve the following problems

1. \( T(n) = T\left(\frac{9n}{10}\right) + n \)

2. \( T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n} \)

3. \( T(n) = 3T\left(\frac{n}{3} + 5\right) + \frac{n}{2} \)
3 Linear Algebra [2+3+3+2]

Give the definition of rank of a matrix. Find the rank and eigenvalues of the matrix B, defined as

\[ B = \begin{bmatrix} 4 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & 3 & 4 \end{bmatrix} \]

If the matrix B has 0 as an eigenvalue, do you think B is invertible? (justify your claim)