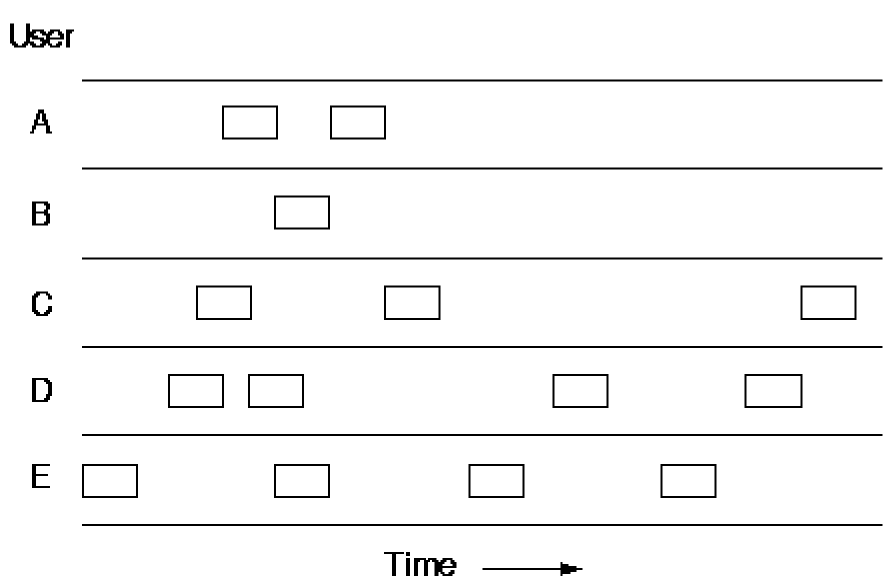
Notes on the efficiency of ALOHA

ALOHA was invented at the University of Hawaii by Norman Abramson in the 1970's. The idea is applicable to systems in which uncoordinated users are competing for a single channel (shared resource). Since there is competition for a single resource this kind of system is known as a contention system. There are two types of ALOHA - Pure ALOHA (no global time synchronization) and Slotted ALOHA (global time synch. required).

Pure Aloha

ALOHA permits users to transmit any time they feel like. Collisions will occur and therefore colliding frames will be destroyed. However, if feedback is available on the destruction, then users will be made aware of their frames have not been transmitted (received) successfully. In a LAN, the feedback is immediate while on a satellite-based transmission the delay is 270msec. If the frame is destroyed the sender waits a random time and transmits again.

As it turns out, the frame lengths in ALOHA are all identical since the throughput can be maximized that way. The figure below shows frame generation:



Whenever two (or more) frames try to occupy the channel at the same time, there will be a collision and all such frames will be garbled. Even if the first bit of a frame overlaps the last frame of a nearly finished frame this garbling will occur.

Based on the above descriptions we can ask the following question – "what percent of all transmitted frames escape collisions?" If we can answer this question we are effectively answering the question of how efficient pure ALOHA is.

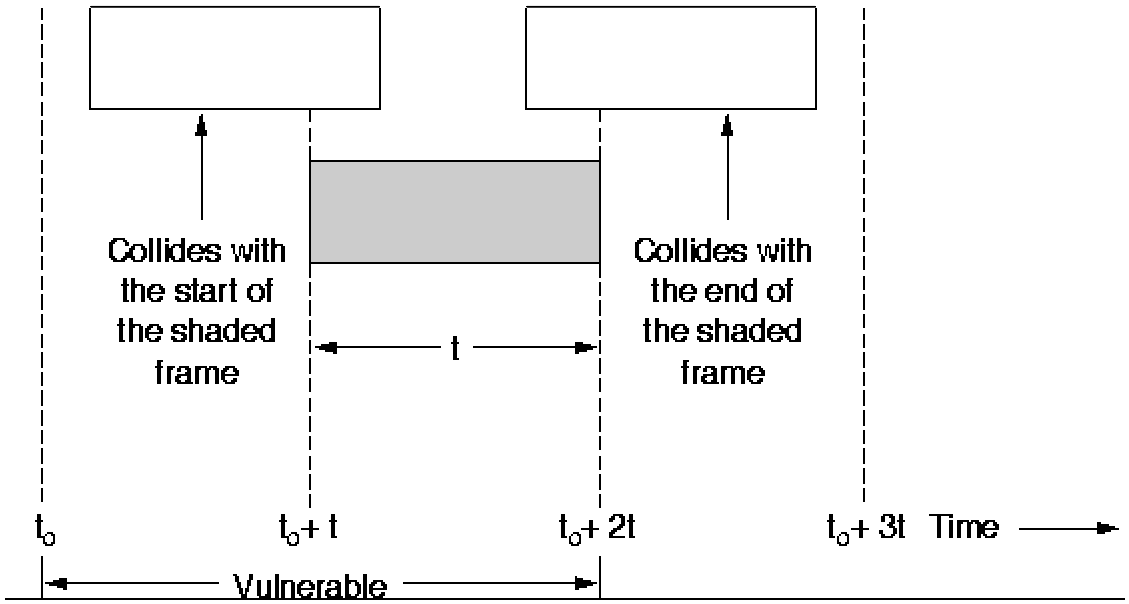
Consider an infinite number of transmitters. Users of these transmitters are in one of two states, typing or idling. Initially all users are in the typing state. When a message is composed, the user sends it and stops typing – waiting for a response. The transmitter transmits the frame containing the message and checks the channel for success (no collision) or failure (collision). If successful, the user goes back to typing otherwise the user waits till a retransmission of the frame succeeds.

We define the "**frame time**" as the amount of time to transmit a fixed length frame (= frame length/bit rate of channel). Assume that the population of transmitters generates new frames according to a Poisson distribution with mean N frames per frame time. (More on the Poisson below). If $N > 1$ then the transmitters are generating frames at a rate that is greater than the channel capacity and therefore nearly every frame will suffer a collision.

Given that $N>1$ is not a good idea, we would like $0 < N < 1$. Stations generate retransmissions due to collisions. Assume that the probability of k transmission attempts per frame time (old transmissions and new transmissions combined) is also Poisson with mean G per frame time. Then $G \geq N$. At low load, ($N \cong 0$), there will be a small # of collisions and therefore a correspondingly low number of retransmissions. Therefore $G \cong N$. At high load we will have many collisions and therefore $G > N$.

Define P_0 = Fraction of attempted frames that are transmitted successfully = S/G = probability that the frame does not suffer a collision. Or, $S = G P_0$ is the throughput per frame time.

At this time we need to define the **vulnerable period** – we do this with the aid of a diagram.



If the shaded frame has been sent (above), under what conditions will the shaded frame arrive intact? Let t be the time to send a frame – if another user has sent a frame between t_0 and $t_0 + t$, the end of the frame will collide with the shaded one. Similarly on the right side of the shaded frame – any frame that began between $t_0 + t$ and $t_0 + 2t$ will have an overlap with the end of the shaded frame. Therefore the **vulnerable period** is 2t – or two frame times.

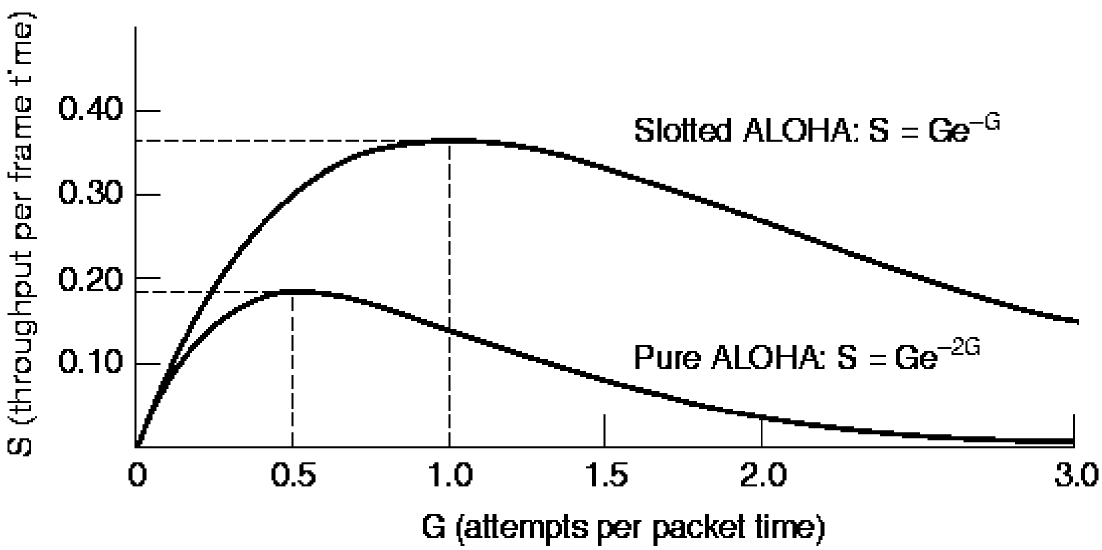
The probability that k frames are generated in one frame time is Poisson (note we have defined G before):

Pr[k] = [G**k x exp(-G)]/k! (1)

Where ** indicates the power operation and exp(-G) is e raised to power –G and k! represents factorial k. Note that formula (1) is the definition of a Poisson distribution. Therefore Pf[k=0] = exp(-G), i.e., the probability of zero frames in one frame time is exp(-G).

In an interval 2 frame times long (the vulnerable period), the mean number of frames is 2G. Therefore the probability that other traffic is generated during the entire vulnerable period is $P_0 = \exp(-2G)$. But $S = G P_0$ (derived previously). Hence $S = G \exp(-2G)$ is the throughput per frame time.

Throughput of pure ALOHA is shown below:



The maximum throughput occurs at $G = 0.5$ and the value of throughput is 0.18 (approximately), i.e., 82% of frames end up in collisions and are therefore lost.

Also view the slides placed on the professors website for details on how the throughput of slotted ALOHA is derived.