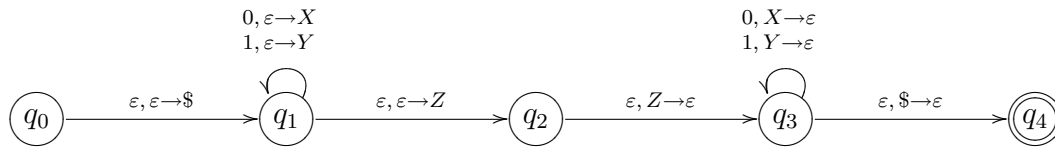


Here's a PDA for the language $\{ ww^R \mid w \in \{0, 1\}^* \}$:



Now, we construct a context free grammar $G = (V, \Sigma, \Gamma, R, S)$ from the PDA above. There are 5 states in the PDA, so we'll have 26 variables in G — 1 start symbol and a variable A_{ij} for each pair of states q_i and q_j . Thus the variables of G are: $S, A_{00}, A_{01}, A_{02}, A_{03}, A_{04}, A_{10}, A_{11}, A_{12}, A_{13}, A_{14}, A_{20}, A_{21}, A_{22}, A_{23}, A_{24}, A_{30}, A_{31}, A_{32}, A_{33}, A_{34}, A_{40}, A_{41}, A_{42}, A_{43}, A_{44}$.

Now, we list all the rules (productions) in G . Since q_0 is the start state of the PDA and q_4 is the unique accepting state, we have the special rule:

$$S \rightarrow A_{04}$$

Also, for each state q_i of the PDA, we add the rule $A_{ii} \rightarrow \epsilon$. So, we have:

$$A_{00} \rightarrow \epsilon$$

$$A_{11} \rightarrow \epsilon$$

$$A_{22} \rightarrow \epsilon$$

$$A_{33} \rightarrow \epsilon$$

$$A_{44} \rightarrow \epsilon$$

Next, for each triplet of states q_i, q_j and q_k , we add the rule $A_{ij} \rightarrow A_{ik}A_{jk}$. This gives us another $5^3 = 125$ rules in G :

$$A_{00} \rightarrow A_{00}A_{00} \mid A_{01}A_{10} \mid A_{02}A_{20} \mid A_{03}A_{30} \mid A_{04}A_{40}$$

$$A_{01} \rightarrow A_{00}A_{01} \mid A_{01}A_{11} \mid A_{02}A_{21} \mid A_{03}A_{31} \mid A_{04}A_{41}$$

$$A_{02} \rightarrow A_{00}A_{02} \mid A_{01}A_{12} \mid A_{02}A_{22} \mid A_{03}A_{32} \mid A_{04}A_{42}$$

$$A_{03} \rightarrow A_{00}A_{03} \mid A_{01}A_{13} \mid A_{02}A_{23} \mid A_{03}A_{33} \mid A_{04}A_{43}$$

$$A_{04} \rightarrow A_{00}A_{04} \mid A_{01}A_{14} \mid A_{02}A_{24} \mid A_{03}A_{34} \mid A_{04}A_{44}$$

$$A_{10} \rightarrow A_{10}A_{00} \mid A_{11}A_{10} \mid A_{12}A_{20} \mid A_{13}A_{30} \mid A_{14}A_{40}$$

$$A_{11} \rightarrow A_{10}A_{01} \mid A_{11}A_{11} \mid A_{12}A_{21} \mid A_{13}A_{31} \mid A_{14}A_{41}$$

$$A_{12} \rightarrow A_{10}A_{02} \mid A_{11}A_{12} \mid A_{12}A_{22} \mid A_{13}A_{32} \mid A_{14}A_{42}$$

$$A_{13} \rightarrow A_{10}A_{03} \mid A_{11}A_{13} \mid A_{12}A_{23} \mid A_{13}A_{33} \mid A_{14}A_{43}$$

$$A_{14} \rightarrow A_{10}A_{04} \mid A_{11}A_{14} \mid A_{12}A_{24} \mid A_{13}A_{34} \mid A_{14}A_{44}$$

$$A_{20} \rightarrow A_{20}A_{00} \mid A_{21}A_{10} \mid A_{22}A_{20} \mid A_{23}A_{30} \mid A_{24}A_{40}$$

$$A_{21} \rightarrow A_{20}A_{01} \mid A_{21}A_{11} \mid A_{22}A_{21} \mid A_{23}A_{31} \mid A_{24}A_{41}$$

$$A_{22} \rightarrow A_{20}A_{02} \mid A_{21}A_{12} \mid A_{22}A_{22} \mid A_{23}A_{32} \mid A_{24}A_{42}$$

$$A_{23} \rightarrow A_{20}A_{03} \mid A_{21}A_{13} \mid A_{22}A_{23} \mid A_{23}A_{33} \mid A_{24}A_{43}$$

$$A_{24} \rightarrow A_{20}A_{04} \mid A_{21}A_{14} \mid A_{22}A_{24} \mid A_{23}A_{34} \mid A_{24}A_{44}$$

$$\begin{aligned}
A_{30} &\rightarrow A_{30}A_{00} \mid A_{31}A_{10} \mid A_{32}A_{20} \mid A_{33}A_{30} \mid A_{34}A_{40} \\
A_{31} &\rightarrow A_{30}A_{01} \mid A_{31}A_{11} \mid A_{32}A_{21} \mid A_{33}A_{31} \mid A_{34}A_{41} \\
A_{32} &\rightarrow A_{30}A_{02} \mid A_{31}A_{12} \mid A_{32}A_{22} \mid A_{33}A_{32} \mid A_{34}A_{42} \\
A_{33} &\rightarrow A_{30}A_{03} \mid A_{31}A_{13} \mid A_{32}A_{23} \mid A_{33}A_{33} \mid A_{34}A_{43} \\
A_{34} &\rightarrow A_{30}A_{04} \mid A_{31}A_{14} \mid A_{32}A_{24} \mid A_{33}A_{34} \mid A_{34}A_{44}
\end{aligned}$$

$$\begin{aligned}
A_{40} &\rightarrow A_{40}A_{00} \mid A_{41}A_{10} \mid A_{42}A_{20} \mid A_{43}A_{30} \mid A_{44}A_{40} \\
A_{41} &\rightarrow A_{40}A_{01} \mid A_{41}A_{11} \mid A_{42}A_{21} \mid A_{43}A_{31} \mid A_{44}A_{41} \\
A_{42} &\rightarrow A_{40}A_{02} \mid A_{41}A_{12} \mid A_{42}A_{22} \mid A_{43}A_{32} \mid A_{44}A_{42} \\
A_{43} &\rightarrow A_{40}A_{03} \mid A_{41}A_{13} \mid A_{42}A_{23} \mid A_{43}A_{33} \mid A_{44}A_{43} \\
A_{44} &\rightarrow A_{40}A_{04} \mid A_{41}A_{14} \mid A_{42}A_{24} \mid A_{43}A_{34} \mid A_{44}A_{44}
\end{aligned}$$

Finally, for each stack symbol $t \in \Gamma$, we look for a pair of states, q_i and q_j , such that the PDA in state q_i can read some input $a \in \Sigma_\varepsilon$ and push t on the stack and in state q_j can read some input $b \in \Sigma_\varepsilon$ and pop t off the stack. In that case, we add the rule

$$A_{ik} \rightarrow aA_{\ell j}b$$

where $(q_\ell, t) \in \delta(q_i, a, \varepsilon)$ and $(q_k, \varepsilon) \in \delta(q_j, b, t)$. This gives us the remaining rules of G :

$$\begin{aligned}
A_{04} &\rightarrow A_{13} \\
A_{13} &\rightarrow A_{22} \\
A_{13} &\rightarrow 0A_{13}0 \\
A_{13} &\rightarrow 1A_{13}1
\end{aligned}$$