

Brown eggs problem from Spring 2009 final exam.
Question 4e.

Recall that we have 60 eggs, of which 18 are brown.
We pick 5 eggs randomly, without replacement.

What is the expected # of brown eggs?

Let's use linearity of expectations. Even though the brown eggs are not distinguishable in the problem, we will nevertheless number them 1 through 18.

For $1 \leq i \leq 18$, let X_i be a random variable such that

$$X_i = \begin{cases} 1 & \text{if the } i\text{th brown egg is one of the 5 picked} \\ 0 & \text{otherwise.} \end{cases}$$

Then $X = X_1 + X_2 + \dots + X_{18}$ is the number of brown eggs picked.

It is easy to compute $E[X_i]$:

$$E[X_i] = 0 \cdot \text{Prob}[i\text{th brown egg not picked}] + 1 \cdot \text{Prob}[i\text{th brown egg picked}]$$

$$= \text{Prob}[i\text{th brown egg is picked}]$$

$$= \frac{1}{60} + \frac{59}{60} \cdot \frac{1}{59} + \frac{59}{60} \cdot \frac{58}{59} \cdot \frac{1}{58} + \frac{59}{60} \cdot \frac{58}{59} \cdot \frac{57}{58} \cdot \frac{1}{57} + \frac{59}{60} \cdot \frac{58}{59} \cdot \frac{57}{58} \cdot \frac{56}{57} \cdot \frac{1}{56}$$

$$= \frac{1}{60} + \frac{1}{60} + \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{5}{60} = \frac{1}{12}$$

meaning it is one of the 5 picked or not one of the 5 picked.

$= 1/60$ numbers cancel nicely

picked 1st

not picked first

picked second

not 1st

not 2nd

not 3rd

not 4th

picked 5th

(2)

By linearity of expectations,

$$E[X] = E[X_1 + X_2 + \dots + X_{18}]$$

$$= E[X_1] + E[X_2] + \dots + E[X_{18}]$$

$$= 18 \cdot \frac{1}{12} = 1.5$$