Exam 1 Spring 1998 CMSC203 Discrete Structures

Symbols: $\mathbf{N}$ denotes the Natural Numbers, $\mathbf{Z}$ denotes the Integers, $\mathbf{Q}$ denotes the Rational Numbers, $\mathbf{I}$ denotes the Irrational Numbers, and $\mathbf{R}$ denotes the Real Numbers, $\mathcal{P}(A)$ is the Power Set of a set A.

1. Circle T if the statement is true or F if the statement is false.

T F $\mathbf{I} \cap \mathbf{R} = \varnothing$.

T F $\{\varnothing\} \in \mathcal{P}(\{\varnothing\})$.

T F The inverse of the statement, ***Some prime numbers are odd*** is the statement, ***All prime numbers are not odd***.

T F The following is a valid argument:

$$\begin{array}{l} \sim p \wedge \sim q \\ \sim p \rightarrow (r \vee \sim s) \\ s \rightarrow q \\ \underline{t \rightarrow r} \\ \therefore t \end{array}$$

T F If A = {11,1}, then A × A = {(11,11), (11,1), (1,11), (1,1)}.

T F If $\Sigma = \{a, b, c\}$ is an alphabet, then $abcabc \in \Sigma^2$.

T F The set of the integers and the set of their reciprocals partition the set of rationals.

T F If A, B, and C are sets, then $[(A \cup B) \cap C]^c = A^c \cup B^c \cap C^c$.

T F gcd(1,0) = 0.

T F The Disjunctive Normal Form of the Boolean Polynomial $F(x, y, z) = x'z$ is $F(x, y, z) = x'yz + xyz'$.

2. Use the Euclidean Algorithm to find gcd(466,112).

3. Given the informal language statement: ***Some integers are perfect squares***

a. Rewrite the statement in formal language as a Universal Conditional.

b. Find the negation of the statement (in either formal or informal language).

4. Show, **without** using truth tables, that $\sim[p \wedge (\sim q \vee \sim r)] \equiv (p \rightarrow q) \wedge (p \rightarrow r)$.

5 a. Find the Disjunctive Normal Form of the Boolean function whose truth table is:

w	x	y	z	$F(w,x,y,z)$
1	1	1	1	0
1	1	1	0	0
1	1	0	1	1
1	1	0	0	0
1	0	1	1	0
1	0	1	0	0
1	0	0	1	1
1	0	0	0	1
0	1	1	1	0
0	1	1	0	0
0	1	0	1	0
0	1	0	0	1
0	0	1	1	1
0	0	1	0	0
0	0	0	1	0
0	0	0	0	0

b. Find the truth table of the Boolean function: $F(x,y,z) = xy'$

6. For the sets A = {2,3,4,5}, B = {1,2,3} and Y = {0,1}, verify that $(A - B) \times Y = (A \times Y) - (B \times Y)$

7. Prove 2 of the 4 theorems below, using the indicated method:

Theorem 1: For all integers, n, if $3n$ is odd, then n is odd. (By the Method of Contraposition)

Theorem 2: If $\sqrt{2}$ is irrational, then $\frac{1 + 3\sqrt{2}}{2}$ is irrational.

Theorem 3: If A, B, and C are sets, then $(A \cup B) - (C \cap D) = (A - C) \cup (A - D) \cup (B - C) \cup (B - D)$.

Theorem 4: If m,n are odd integers, then $m^2 - n^2$ is even.