Spring 1994 DISCRETE STRUCTURES - CMSC203 FINAL - Part One

1. Given the following LOGIC statement: $((A \wedge B) \vee \neg(\neg A \vee \neg B) \wedge C) \rightarrow D$
(a) Simplify the statement.
(b) Fill in the truth table for the statement:
(c) Viewing this statement as a Boolean Polynomial (or Combinatorial Circuit), draw its circuit diagram.

2. Given the statement: $x^2 > |x|, if |x| > 1, \forall x \in$ Z, fill in its:
NEGATION, **CONTRAPOSITIVE, CONVERSE, INVERSE**

3. A sequence $U_0, U_1, \ldots, U_n$ is defined recursively as follows: $U_0 = 2$, $U_1 = 2$, $U_k = U_{k-2} \bullet U_{k-1}$, for all k > 1. (a) (6 pts) Find an closed form formula for the sequence. (HINT: Recall the Fibonacci sequence 1,1,2,3,5,8,... satisfies $F_k = F_{k-1} + F_{k-2}$.)

(b) Prove inductively that it satisfies the definition above.

Part Two

4. Let f and g be binary relations on Z_{12} defined by:
$f = \{(x,y) \in Z_{12} \times Z_{12} \mid y \equiv 3x+1 \bmod 12\}$ and $g = \{(x,y) \in Z_{12} \times Z_{12} \mid y \equiv 5x+1 \bmod 12\}$.
(a) Draw the directed graph of the relations f and g.
(b) Answer ***yes*** or ***no*** to the following questions, and briefly explain your answer based on interpretation of the graph.
Is f a function? Is f injective (1-1)? Is f surjective (onto)? Is f bijective (1-1 Corr.)?
(c) Using the definitions of injective and surjective, carefully prove or disprove that g is bijective.
(d) Let R be the relation, $R = \{(x,y) \in Z_{12} \times Z_{12} \mid f(x) = f(y)\}$, where f is as defined above. Prove, in detail, that R is an equivalence relation on Z_{12}.
(e) List the equivalency classes in Z_{12} induced by R.

5. Consider the following program, DUP, which lists all duplicate elements of an array:

```
BEGIN DUP(A,N)
    FOR J FROM 0 TO (N-2)
        FOR K FROM (J+1) TO (N-1)
            IF (A[J] = A[K]) THEN PRINT("Duplicate found at ",J," and ",K)
        NEXT K
    NEXT J
END
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Let T(n) denote the number of times the "IF.THEN ." statement is evaluated during the execution of DUP.
(a) Give a closed form expression for T(N) in terms of N. Simplify your answer as a sum of terms on decreasing orders of growth. Carefuly justify your answer and draw a box around your final answer.
(b) Find the "Big Oh" for T(N)

6. Let A, B, and C be any sets with A = B ∪
which partition A.
(a) Express the cardinality of A in terms of the sets B and C.

7. Prove: If F:A →
equal to the cardinality of B.
8. (a) Complete the statement of the Binomial Theorem: $x \quad y^{n}$

(b) Calculate (2.1) = (2 + 0.1) using the Binomial Theorem.

$\binom{n}{} \quad \binom{n-}{1} + \binom{n-}{}$

9. How many 20-digit numbers have exactly five 3’s, six 4’s and seven 5’s?

hosts given by the property that it must reside dormant on a host before it can spread to
two hosts
to a new host, it stays active on the original as well.

are active or dormant on any given day.

dormant
how many total viruses will there be in 100 days? (Obtain an expression for the number but don’t