DISCRETE STRUCTURES - CMSC 203 - FINAL EXAM - SPRING 2002

1. Fill in the blank(s).

(a) $\frac{(39 \text{ div } 6)[GCD(1280,324)]}{39 \bmod 6}$ = _____.

(b) $n + (n-1)/2! + (n-2)/3! + (n-3)/4! + \ldots + 1/n! = \sum_{i=__}^{__} _____$

(c) $(\{1,3,4,7,9\} - \{2,3,4,5\}) \cup (\{4,5,6,7\} \cap \{1,2,3,4,5,6\})$ = _____

(d) H(1110001,1001001) = _____ (H is the Hamming Distance Function)

(e) If $\Sigma = \{x,y,z\}$, then $|\Sigma^4|$ = _____.

(f) DeMorgan's Law states that for any sets A and B, $(A \cap B)^c$ = _____.

(g) For finite sets A and B, the Inclusion/Exclusion Rule says: $|A \cup B|$ = ____+____ – _____.

(h) If $\Sigma = \{0,1\}$, then $\Sigma \times \Sigma$ = _____.

(i) If $\Sigma = \{0,1\}$, then Σ^2 = _____.

(j) Let F:{1,2,3,4} → {*a*,*b*,*c*,*d*} be a one-to-one corresp. Then F = {(1,*b*), (2,*d*), (3,*a*), _____}

(k) If G:{*x*,*y*,*z*} → B is onto, and G = {(*x*,1), (*y*,2), (*z*,0)}, then B = _____.

(l) If $s_n = 3s_{n-1} + 4s_{n-2}$ with $s_0 = 1$ and $s_1 = -1$, then s_4 = _____.

(m) If f:**R** → **R** is defined by $f(x) = (2x - 7)/3$, then $f^{-1}(x)$ = _____.

2. Use the Euclidean Algorithm to find GCD(1280, 324).

3. Find the Disjunctive Normal Form of $F(x,y,z) = (x + yz')(xz')$.

4. Find the truth table of the statement $[p \vee (q \wedge \sim r)] \rightarrow \sim q$.

5. Given the statement: ***For all n ∈ Z, if n is prime, then n is not divisible by 2***
its negation is _____.

6. Give an example of Universal Modus Tollens:
Major Premise: _____
Minor Premise: _____
Conclusion: _____

7. Prove: If *p* is prime, then for any integer, *n*, for which *p* divides n^2, it follows that *p* divides *n*.

8. Prove that $\sqrt{7}$ is irrational.

9. Determine whether the relation R = {(*x*, *y*) | *x*, *y* are Integers and $x^2 = y^2$} is REFLEXIVE, SYMMETRIC, TRANSITIVE, or none of these.

10. Let A = {$a_1, a_2, \ldots, a_k$} and let $b \notin$ A. Show f: $\wp$(A) → $\wp$(A∪{b}) defined by: f(S) = S ∪ {*b*}, for S ⊆ A, is a one-to-one correspondence. (Note: $\wp$(A) is the Power Set of A)

11. Use Mathematical Induction to show: If A has *n* elements, then A has 2^n subsets.

12. A youth group is made up of 22 boys and 30 girls. How many different ways can:
(a) they form a line? **(b)** they form a circle? **(c)** they pick 5 boys and 7 girls to race each other?
(d) they pick 4 kids to run a relay race (with starter being distinct from second runner, etc.)?

13. I have a large collection of $1 coins and want to distribute them into 15 piles so that each pile has at least 2 coins, and the total value of the piles is $75. How many ways can I do this?

14. How many orderings are there of the letters of the word *ORGANIZATION* ?

15. Find the characteristic poly. for the recurr. relation: $s_n = s_{n-1} - 2s_{n-2} + 3s_{n-3} - 4s_{n-4} + 5s_{n-5}$

16. (a) Find the General Solution to: $s_n + 11s_{n-1} + 28s_{n-2} = 0$.
(b) If a recurrence relation has the General Solution $s_n = A(-5)^n + B(7)^n$, find its Particular Solution when $s_0 = -3$ and $s_1 = 0$.