CMSC 203 - Discrete Structures - Spring 2002 - Exam 3

1. Suppose I have a collection of 24 Math books, 25 Chemistry books, 26 Biology books, and 30 Geology books.

a. How many ways can I arrange all these books on a shelf if all the books are distinct?

b. How many ways can I arrange all these books on a shelf if all the books are **distinct** and I want the books of each category to be adjacent?

c. How many ways can I select 15 books if all the books of each type are **identical**?

d. How many ways can I select 20 books if all the books of each type are **identical** and I want at least 3 of each type?

e. How many ways can I select 20 books if all the books are **distinct** and I want 3 of each type?

2. Suppose 15 people go to eat at a restaurant.

a. How many ways can they arrange themselves around a round table?

b. How many ways can they arrange themselves around a round table if a certain pair of people cannot sit adjacent to one another?

3. Verify: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

4. Fill in the blanks for the first 10 rows of Pascal's Triangle:

Row 0: 1

Row 1: 1 1

Row 2: 1 _____ 1

Row 3: 1 _____ _____ 1

Row 4: 1 _____ _____ _____ 1

Row 5: 1 _____ _____ _____ _____ 1

Row 6: 1 _____ _____ _____ _____ _____ 1

Row 7: 1 _____ _____ _____ _____ _____ _____ 1

Row 8: 1 _____ _____ _____ _____ _____ _____ _____ 1

Row 9: 1 _____ _____ _____ _____ _____ _____ _____ _____ 1

5. Find the next 5 terms in the recurrence relation: $s_n = (s_{n-1})(s_{n-3}) - (s_{n-2})^2$ when $s_0 = 1$, $s_1 = 0$ and $s_2 = (-1)$

6. Use the Method of Iteration to find a general solution to the recurrence relation:

$$s_n = 5s_{n-1} + 3, \text{ when } s_0 = 1.$$

7. Find the general solution to the recurrence relation whose characteristic polynomial has roots 3,3,3,3,3,3,(−2),(−2),(−2),(−2).

8. Find the General Solution to the 2nd order, linear, homogeneous recurrence relation with constant coefficients: $s_n = s_{n-1} + 56s_{n-2}$.

9. Find the particular solution to the recurrence relation whose general solution is

$$s_n = A4^n + B(-3)^n, \text{ when } s_0 = 4 \text{ and } s_1 = 51.$$