CMSC203 - FINAL EXAM - Fall 1999

1. Circle T if the statement is true or F if the statement is false.

T F $\varnothing$ is a subset of all sets.

T F A conditional statement and its converse are logically equivalent.

T F If A and B are non-empty sets, then $A \times B = B \times A$.

T F If p, q, and r are statements, then: $p \to \sim(q \wedge \sim r) \equiv (p \wedge q) \to r$.

T F The negation of a universal conditional is an existential disjunction.

T F The set of irrational numbers are closed under addition.

T F $\{(1,1), (3,3), (5,5), (7,7)\}$ is an equivalence relation on $\{1,3,5,7\}$.

T F If f:A $\to$ B is an one-to-one function, then $|A| = |f(A)|$.

T F If d is the density function on binary strings, then $d(\Sigma^4) = \{1, 2, 3, 4\}$.

T F $C(10,5) = C(9,4) + C(9,5)$.

T F The general solution of a LHSORRCC whose characteristic polynomial has the sole root $x = 5$ is $s_n = (C + Dn)5^n$, where C and D are arbitrary constants.

2. Find the Disjunctive Normal Form of the Boolean Polynomial: $F(x,y,z) = x + y'z$.

3. Negate: *Every integer greater than 0 has a square root greater than 0*.

4. Graph the relation R on $\{0,1,2,3,4\}$ given by $R = \{(x,y) \mid 2x + y < 7\}$.

5. What partition of **Z** is induced by the equivalence relation "congruence mod 5" ?

6. Given the functions f and g, find $f^{-1} \circ g^{-1}$ when:

$f = \{(1,4), (2,3), (3,2), (4,5), (5,1)\}$ and $g = \{(1,4), (2,2), (3,5), (4,1), (5,3)\}$.

7. What is the general solution of the recurrence relation $s_n = 8s_{n-1} + 33s_{n-2}$?

8. What is the particular solution to the recurrence relation whose general solution is $s_n = (A + Bn)4^n$ when $s_0 = 5$ and $s_1 = 8$.

9. Use the Euclidean Algorithm to find GCD(572,221).

10. How many distinct boxes of candy can be created by a candy store that sells 20 different types of candy and a box must contain 50 pieces with at least one of each type?

11. How many ways can I order 10 people around a circular table if a certain pair of people cannot sit next to one another?

12. Prove 3 of the 6 statements below by the indicated method.

(a) If a, b, x, and y are positive integers with $a = bx + y$, then $GCD(a,b) = GCD(b,y)$.

(b) If A is a set with n elements, then A has 2^n subsets. (Weak Induction)

(c) $\sqrt{2}$ is irrational. (Contradiction)

(d) The relation "congruence mod 5" is an equivalence relation on **Z**.

(e) Every integer greater than 1 has a prime factor. (Strong Induction)

(f) The function $f = \{(x,y) \mid x,y \in \mathbf{R} \text{ and } 2y = 3x - 4\}$ is a bijection.