CMSC 203 - Fall 2002 - Exam 2

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** There are C(16,5) 16-bit binary words with five 1's and eleven 0's.

T **F** If A, B, C, and D are pairwise disjoint sets, $|A \cup B \cup C \cup D| = |A| + |B| + |C| + |D|$.

T **F** The First and Second Principles of Mathematical Induction are logically equivalent.

T **F** Every recursively defined sequence must take on successively larger values.

T **F** If hard-working students pass CMSC203 and Paul passed CMSC203, then we can conclude logically that Paul is hard-working.

T **F** The following is a valid argument:

$$\begin{array}{l} \neg p \wedge q \\ \neg r \rightarrow s \\ (r \wedge t) \rightarrow \neg q \\ t \vee p \\ \text{--------------} \\ \therefore s \end{array}$$

T **F** Allowing the top to be the 0th row, the sum of elements of the 201st row of Pascal's Triangle is 2^{201}.

T **F** There are 11! distinct orderings of the letters of the word MATHEMATICS.

T **F** For every recursive algorithm, there is an equivalent iterative algorithm.

2. Circle **V** for valid or **I** for invalid with respect to the following arguments:

V **I** All dogs run fast and Zeke runs fast, therefore Zeke is a dog.

V **I** All dogs run fast and Zeke runs slow, therefore Zeke is not a dog.

V **I** All dogs run fast and Zeke is not a dog, therefore Zeke runs fast.

V **I** All dogs run fast and Zeke is a dog, therefore Zeke runs fast.

3. On a class field trip, a class of 10 boys and 12 girls must form a single-file line. **(a)** How many distinct ways can they form this line? **(b)** How many distinct ways can they form this line of all the kids of the same gender must stay grouped together? **(c)** How many ways can they form this line if a certain pair of boys cannot be next to one another?

4. Suppose I have a collection of 200 pennies. **(a)** How many distinct ways can I create 10 piles of these pennies? **(b)** How many distinct ways can I create 10 piles if each pile must have at least 5?

5. Prove one of the two Theorems below using Mathematical Induction.

Theorem 1: For all integers $n \geq 0$ and $a \neq 0,1$, $\displaystyle\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

Theorem 2: Assuming Pascal's Identity, then the sum of the entries of Row n of Pascal's Triangle is 2^n.

6. Use the Methods of Valid Arguments to obtain the indicated conclusion.

Premises: Paul does his homework or Paul watches TV.
If Paul does not watch TV, then Paul gets good grades.
If Paul gets good grades and Paul gets a good job, then Paul does his homework.
Paul does not watch TV and Paul makes the honor roll.

Conclusion: Therefore, Paul gets a good job.

7. Prove one of the two Theorems below by either Contradiction or Contraposition.

Theorem 1: There is no greatest integer.

Theorem 2: For all integers n, if n^2 is odd, then n is odd.