CMSC203 - Discrete Structures - Final Examination - Spring 1999

Part One

1. Using the Properties of Sets, show that if A and B are sets, then A \cap (A \cup B) = A.
2. Find the Power Set of \{1,\{1\}\}.
3. Show that if p,q, and r are statements, then p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r).
4. Show the following is a valid argument:
   \begin{align*}
p & \rightarrow q \\
r & \lor s \\
r & \rightarrow t \\
\neg q \\
s & \rightarrow p \\
\therefore t
\end{align*}
5. Find the Disjunctive Normal Form of the Boolean Polynomial F(x,y,z) = xy' + z
6. What is the negation of the statement: All integers that are even are divisible by 2.
7. What is universal modus tollens? Draw a picture to illustrate it.
8. Prove one of the following statements:
   a. The square root of an irrational number is irrational.
   b. If n is an odd integer, then 8 \mid (n^2 - 1).
9. Use the Division Algorithm to find 37 MOD 5 and 37 DIV 5.
10. Use the Euclidean Algorithm to find GCD(274,136).

Part Two

11. Rewrite \sum_{i=1}^{10} i^{10-i} as a summation from 0 to 9.
12. Prove one of the two statements by Mathematical Induction:
   a. If n is a positive integer, \sum_{i=0}^{n} 7^i = \frac{7^{n+1} - 1}{6}.
   b. Every positive integer has a binary representation.
13. Draw the directed graph of the relation R = \{(a,b) \mid a,b \in \{1,2,3,4,5,6\} \text{ and } a + b \text{ is even}\}
14. Show the relation Congruence Modulo 7 is an Equivalence Relation.
15. If f: \{1,2,3,4\} \rightarrow \{5,7,9,11\} and g: \{1,3,5,7,9,11\} \rightarrow \{2,10,26,50,82,122\} are given by f(x) = 2x + 3 and g(x) = x^2 + 1, calculate (g \circ f)^{-1}
16. Let f: \mathbb{Z} \rightarrow \mathbb{Z}_{even} be given by f(x) = 2x + 4. Show f is a bijection.

Part Three

17. How many 8-character license plates made up of 26 letters and 10 digits begin with “A1” and end with “Z9”?
18. How many ways can 2 pennies, 3 nickels, 4 dimes, and 5 quarters be ordered in a line?
19. How many integer solutions are there to the equation: a + b + c + d + e + f = 50 with
   \begin{align*}
   a & \geq 2, \\
b & \geq 4, \\
c & \geq 6, \\
d & \geq 1, \\
e & \geq 3, \text{ and } f \geq 5?
   \end{align*}
20. TRUE or FALSE? C(7,0) + C(7,1) + \ldots + C(7,7) = 128.
21. Use the Iteration Method to solve s_n = 7s_{n-1} + 1, when s_0 = 1.
22. Find s_{152637} given s_n = 3s_{n-1} + 28s_{n-2} and s_0 = 2 and s_1 = 5.