Discrete Structures - Spring 1999 - Exam 2

Symbols: **N** denotes the Natural Numbers, **Z** denotes the Integers, **Q** denotes the Rational Numbers, and **R** denotes the Real Numbers.

1. Circle T if the statement is true or F if the statement is false.

T F If A is a set, then A×A is a function.

T F If A is a set, then A×A is an equivalence relation.

T F If the sets A, B, C, and D partition a set X, then the relation R = (A×A) ∪ (B×B) ∪ (C×C) ∪ (D×D) is an equivalence relation on X.

T F If f is a 1–1 and ONTO function from A to B, then A = B.

T F If f is a 1–1 function from A to B, then |A| = |f(A)|.

T F If f:A → B and g:B → C are functions, then $(g \circ f)^{-1} = (g^{-1} \circ f^{-1})$.

T F If n is a positive integer, $1 + 10 + 10^2 + ... + 10^{(n-1)} = (10^n - 9)$.

T F The Weak and Strong Forms of Mathematical Induction are equivalent.

T F If H is the Hamming distance function, d is the density function, and 0 is the all-zero string, then H(s,0) = d(s) for all binary strings, s.

T F There cannot exist a 1–1 correspondence between **N** and **Q**.

2. Rewrite: $\frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \ldots + \frac{512}{10 \cdot 11}$ as a summation from 5 to 14.

3. Let R = {(a,b) | a,b ∈ {1,2,3,4} and a/b ≤ 2}. Graph R.

4. Do 1 of the following 2 induction proofs:

Prove: $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

or

Prove: If s_i is a recursively generated sequence given by $s_i = s_{i-1} + s_{i-2}$, with $s_0 = 5$ and $s_1 = 15$, then s_i is divisible by 5 for all $i > 1$.

5. What equivalence relation is induced by the partition {3}, {1,4}, {0,2,5} of the set {0,1,2,3,4,5}? (List out the ordered pairs that make up the equivalence relation)

6. Find the Domain and Image of the function f = {(a,3),(w,12),(e,2),(s,5),(t,3),(g,2),(v,1)}.

Domain = { } Image = { }

7. Let d be the density function on Σ^n, the set of all n-long strings and define the relation R on Σ^n as R = {(s,t) | s,t ∈ Σ^n and d(s) = d(t)}.

(a) Show R is an equivalence relation on Σ^n. (b) Describe the partition of Σ^n induced by R.

8. Prove the function f:**R** → **R** given by $f(x) = 15x + 13$ is a bijection (1-1 and onto).