CMSC 203 - Discrete Structures - Spring 1998 Final Examination

1. Circle **T** or **F** as it applies to the associated statement below:

T **F** The negation of the statement, "Some integers are negative," is "Some integers are positive."

T **F** The following is a valid argument:

$$\begin{array}{l} \sim q \wedge p \\ \sim q \rightarrow (t \vee s) \\ s \rightarrow \sim p \\ \underline{t \rightarrow r} \\ \therefore r \end{array}$$

T **F** If $p \equiv q$, then $p \leftrightarrow q$ is a tautology.

T **F** If A = {1,3}, B = {1,2,5}, and U = {0, 1, 2, 3, 4, 5}, then $(B^c \cup A)^c = \{3\}$

T **F** If A = $\{x, y\}$ and B = $\{a, b\}$, then B × A = $\{(a, x), (a, y), (b, x), (b, y)\}$

T **F** If f :$\{a, b, c, d, e\} \rightarrow \{v, w, x, y, z\}$ is defined as f = $\{(a,x), (b,z), (c,x), (d,w), (e,y)\}$, then f is an ONTO function.

T **F** If f :$\{a, b, c, d, e\} \rightarrow \{v, w, x, y, z\}$ is defined as f = $\{(a,x), (b,z), (c,x), (d,w), (e,y)\}$, then f is an ONE-TO-ONE function.

T **F** If the relation **R** on A = {0, 1, 2, 3, 4} is **R** = $\{(a, b) \mid a, b \in A \text{ and } b \equiv 3a \bmod 5\}$, then **R** = {(0, 0), (1, 3), (2, 1), (3, 4), (4, 2)}

T **F** The relation {(1, 1), (2, 2), (3, 3), (4, 4)} is both SYMMETRIC and ANTI-SYMMETRIC on the set {1, 2, 3, 4}.

T **F** Let S = {0,1}, H(s, t) be the Hamming Distance Function, and define the equivalence relation R = $\{(s, t) \mid s, t \in \Sigma^4 \text{ and } H(s, 0000) = H(t, 0000)\}$. Then [0011] = {0011, 0000}.

T **F** There are $\frac{14!}{4! \cdot 6! \cdot 4!}$ distinct orderings of the letters *abbabccacbcabb*.

T **F** If A, B, and C are sets which partition a set X, then |A| = |X| – |B| – |C|.

T **F** If n and r are positive integers with $n \geq r$, then $P(n, r) = n!C(n, r)$.

T **F** The Characteristic Polynomial of $s_n = s_{n-3} + s_{n-5}$ is $x^5 - x^3 - 1$.

T **F** If a recurrence relation has the General Solution: $s_n = (A + Bn + Cn^2)(3^n)$, then its Characteristic Polynomial is $(x - 3)^3$.

2. Suppose 100 students take Math, Gym, or Physics, 60 students take Math, 50 students take Gym, 45 students take Physics, 25 students take Math and Gym, 15 students take Gym and Physics, and 10 students take all three. How many students take Math and Physics?

3. Find the Boolean Polynomial for a circuit of 4 inputs which outputs a current whenever the last two inputs are the negation of the first two.

4. In informal language (English), what is the negation of the statement:

For some people, if you ask, then they answer.

5. How many different ways can Andrew, Betty, Charles, Diane, Edward, Fay, Gordon, Harriet, Isaac, and June sit around a circular table so that Andrew and Betty never sit next to one another?

6. If $\Sigma = \{0,1\}$, show that there exists a bijective function from Σ^3 to $\Sigma\times\Sigma\times\Sigma$. .

7. How many integer solutions are there to the equation $a + b + c + d + e + f + g = 50$ provided $a \geq 1$, $b \geq 2$, $c \geq 3$, $d \geq 4$, $e \geq 5$, $f \geq 6$, and $g \geq 7$?

8. Given the recurrence relation $s_n = 4s_{n-1} + 21s_{n-2}$, what is s_{999} when $s_0 = 7$ and $s_1 = -1$?

9. Prove ONE of the TWO statements below:

a. If d, n, q, and r are integers with $n = dq + r$, then GCD(n, d) = GCD(d, r).

b. The square root of 2 is an irrational number.

10. Prove ONE of the TWO statements below:

a. $$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

b. If a_0, a_1, a_2, ... is the sequence: $a_0 = 1$, $a_1 = 3$, $a_2 = 7$ with $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, then a_n is odd for all $n \geq 3$.

11. Prove ONE of the TWO statements below:

a. The function f: $\mathbf{R} \to \mathbf{R}$ given by $4y + 3x = 7$ is a bijection.

b. The relation R on $\mathbf{Z}$ given by R = $\{(a, b) \mid a, b \in \mathbf{Z} \text{ and } b \equiv a \bmod 7\}$ is an equivalence relation.