Notation: Let R denote the Real Numbers, and P(A) denote the Power Set of A.

1. (20 pts.) Circle T if the statement is true or F if the statement is false.

   T   F   R \subseteq R \times R.
   T   F   The set \{1,2,3,4\} has 16 subsets.
   T   F   \emptyset \subset P(\emptyset,1) and \emptyset \in P(\{0,1\}).
   T   F   The negation of the statement: All Natural Numbers are positive
   is the statement: No Natural Numbers are positive.
   T   F   [(36 \text{ DIV } 7) - (93 \text{ MOD } 5)] = 2.
   T   F   If \(d \mid (x + y)\) and \(d \mid x\), then \(d \mid y\).
   T   F   If \(A = \{0,1\}\), then \(A \times A \times A = \{000,001,010,011,100,101,110,111\}\).
   T   F   If \(\Sigma = \{0,1\}\), then \(|\Sigma^5| = 32\).
   T   F   The set of even integers and the set of odd integers partition the set of integers.
   T   F   A conditional and its contrapositive are logically equivalent.

2. (6 pts.) Use the Euclidian Algorithm to find gcd(1234,56)

3. (10 pts.) Show, without using truth tables, that \((\neg p \land q) \rightarrow r \equiv p \lor \neg q \lor r\).

4. (4 pts.) Give the converse, inverse, contrapositive, and negation of the universal statement:
   All prime numbers greater than 2 are odd.

5. (10 pts.) Find the Boolean polynomial representing a circuit of four inputs in such a way
   that if the integer value of the inputs is prime, then current flows out of the circuit. (For example, 12 is not prime, and 12 = 1100, so \(f(1100) = 0\))

6. (10 pts.) Show the following is a valid argument:
   \[
   \begin{align*}
   p & \rightarrow (q \land r) \\
   \neg r \quad \text{________} \\
   \therefore \neg p
   \end{align*}
   \]

7. (40 pts.) Prove 2 of the 4 theorems:
   Theorem 1: \((A \cap B)^c = A^c \cup B^c\).
   Theorem 2: The difference of the square of natural number and the square of its successor is odd.
   Theorem 3: There is no largest integer.
   Theorem 4: If \(a, b,\) and \(c\) are integers with \(a = b + c\), then \(\gcd(a,b) = \gcd(b,c)\).