CS 203 - Exam 1 - Spring 1996

Notation: Let R denote the Real Numbers, and P(A) denote the Power Set of A.

1. (20 pts.) Circle T if the statement is true or F if the statement is false.

T F $R \subset R \times R$.

T F The set {1,2,3,4} has 16 subsets.

T F $\varnothing \subset P(\{\varnothing,1\})$ and $\varnothing \in P(\{0,1\})$.

T F The negation of the statement: **All Natural Numbers are positive** is the statement: **No Natural Numbers are positive**.

T F [(36 DIV 7) - (93 MOD 5)] = 2.

T F If $d \mid (x + y)$ and $d \mid x$, then $d \mid y$.

T F If A = {0,1}, then A × A × A = {000,001,010,011,100,101,110,111}.

T F If $\Sigma = \{0,1\}$, then $|\Sigma^5| = 32$.

T F The set of even integers and the set of odd integers partition the set of integers.

T F A conditional and its contrapositive are logically equivalent.

2. (6 pts.) Use the Euclidian Algorithm to find gcd(1234,56)

3. (10 pts.) Show, without using truth tables, that $(\sim p \wedge q) \rightarrow r \equiv p \vee \sim q \vee r$.

4. (4 pts.) Give the converse, inverse, contrapositive, and negation of the universal statement:

All prime numbers greater than 2 are odd.

5. (10 pts.) Find the Boolean polynomial representing a circuit of four inputs in such a way that if the integer value of the inputs is prime, then current flows out of the circuit. (For example, 12 is not prime, and 12 = 1100, so f(1100) = 0)

6. (10 pts.) Show the following is a valid argument:

$$\begin{array}{l} p \rightarrow (q \wedge r) \\ \underline{\sim r \qquad\qquad} \\ \therefore \sim p \end{array}$$

7. (40 pts.) Prove 2 of the 4 theorems:

Theorem 1: $(A \cap B)^c = A^c \cup B^c$.

Theorem 2: The difference of the square of natural number and the square of its successor is odd.

Theorem 3: There is no largest integer.

Theorem 4: If a, b, and c are integers with $a = b + c$, then gcd(a,b) = gcd(b,c).