Discrete Structures - Spring 1995 - Exam 2

1. Circle T if the statement is true or F if the statement is false.
   T  F If A is a set, then AxA is a function.
   T  F If A is a set, then AxA is an equivalence relation.
   T  F If the sets A, B, C, and D partition a set X, then the relation
   \( R = (AxA) \cup (BxB) \cup (CxC) \cup (DxD) \) is an equivalence relation on X.
   T  F If \( f \) is a 1-1 and ONTO function from A to B, then A = B.
   T  F If \( f \) is a 1-1 function from A to B, then |A| = |f(A)|.
   T  F If \( f:A \to B \) and \( g:B \to C \) are functions, then \( (g \circ f)^{-1} = (g^{-1} \circ f^{-1}) \).
   T  F Let DIV denote the INTEGER DIVIDE operation. If a, b, and p are positive integers
   and \( a \equiv b \mod p \), then \([a - p(a \div p)] = [b - p(b \div p)]\).
   T  F If \( H \) is the Hamming distance function, \( d \) is the density function, and \( \emptyset \) is the
   all-zero string, then \( H(s,\emptyset) = d(s) \) for all binary strings, s.
   T  F There cannot exist a 1-1 correspondence between \( \mathbb{N} \) and \( \mathbb{Q} \).

2. Rewrite: \( \frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \ldots + \frac{512}{10 \cdot 11} \) into summation notation.

3. Let \( R = \{(a,b) | a,b \in \{1,2,3,4\} \text{ and } a/b \leq 1\} \). Graph R.

4. Do 1 of the following 2 induction proofs:
   Prove: \( 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3} \).
   or
   Prove: If \( s_i \) is a recursively generated sequence given by \( s_i = s_{i-1} + s_{i-2} \),
   with \( s_0 = 5 \) and \( s_1 = 15 \), then \( s_i \) is divisible by 5 for all \( i > 1 \).

5. List out the ordered pairs that make up the equivalence relation induced by the partition
   \( \{2\}, \{1,3\}, \{0,4,5\} \) of the set \( \{0,1,2,3,4,5\} \)?

6. Find the Domain and Image of the function \( f = \{(a,3),(w,12),(e,2),(s,5),(t,3),(g,2),(v,1)\} \).

7. Let \( d \) be the density function on \( \Sigma^n \), the set of all \( n \)-long strings and define the relation \( R \) on \( \Sigma^n \)
   \( R = \{(s,t) | s,t \in \Sigma^n \text{ and } d(s) = d(t)\} \).
   (a) Show \( R \) is an equivalence relation on \( \Sigma^n \).  (b) Describe the partition of \( \Sigma^n \) induced by \( R \).

8. Prove the function \( f:R \to R \) given by \( f(x) = 5x + 3 \) is a bijection (1-1 and onto).