Discrete Structures - Spring 1994 - Examination 1

1. (20 pts.) Circle **T** if the statement is true or **F** if the statement is false.

T **F** $Z \cap Q = Z$.

T **F** $A \in P(A)$.

T **F** The **negation** of the statement ***All women are strong, all men are handsome, and all children are above average*** is the statement ***Some women are not strong, some men are not handsome, and some children are not above average***.

T **F** The **inverse** of the statement ***If I pass CMSC 203, then I get a good job*** is the statement ***If I do not pass CMSC 203, then I do not get a good job***.

T **F** If A = {*x,y,z*}, then *zz* $\in$ A $\times$ A.

T **F** If Σ = {*x,y,z*}, then *zz* $\in \Sigma^2$.

T **F** Given the set S = {0,1,2,3,4,5,6,7,8,9}, the collection of sets {0,2,4,6,8}, {1,7}, {3}, {9} forms a partition of S.

T **F** (Show work!) $\left[\left[(33 \bmod 7) - \left\lceil \frac{126}{10} \right\rceil\right] \lfloor (-13.2) + 6 \rfloor\right] \bmod 63 = 1$.

T **F** For all integers *x*, if *x* is prime, then *x* is odd.

T **F** A set with 8 elements has 256 subsets.

2. (6 pts.) Given the statement $\boldsymbol{p \rightarrow q}$,

its **converse** is _____;

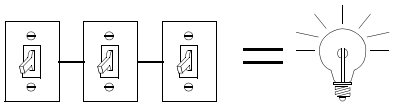
its **inverse** is _____;

its **contrapositive** is _____.

3. (10 pts.) Show, **without using truth tables**, that $\boldsymbol{\sim [\sim (p \wedge \sim q) \rightarrow (p \wedge q)] \equiv \sim p}$.

4. (4 pts.) Write the negation of the universal statement: *For all* $x \in \mathbb{N}$, *if* $x^2 = 4$, *then* $x = 2$.

5. (10 pts.) Find the Boolean polynomial representing a circuit of three switches controlling a light bulb in such a way that if any one switch is flipped, then the light bulb turns on if it is off or turns off if it is on. The switch is configured so that if all three switches are down (off), the light bulb is on.



6. (10 pts.) Using the Properties of Sets, show that: $(A \cap B^c)^c - (A \cap B) = A^c$.

7. (40 pts.) Prove 2 of the 4 theorems:

Theorem 1: $\sqrt{2}$ is irrational.

Theorem 2: If *r* and *s* are rational numbers and $r < s$, then there is a rational number *x* with $r < x < s$.

Theorem 3: For all integers *n*, if n^2 is odd, then *n* is odd.

Theorem 4: For all integers *a,b,c,* and *d,* if $a \mid (b + c + d)$ and $a \mid (b + c)$, then $a \mid d$.