1. Circle T if the corresponding statement is True or F if it is False.
   
   T  F  The sequence \{1, -1, 1, -1, 1, -1 ...\} is an example of an Alternating sequence.
   
   T  F  The Weak and Strong Principles of Mathematical Induction are logically equivalent.
   
   T  F  In general, iterative algorithms use memory more efficiently than their equivalent recursive representations.
   
   T  F  \[1 + 10 + 100 + 1000 + ... + 10^n = 10^{(n+1)} - 1.\]
   
   T  F  \[2 + 4 + 6 + 8 + 10 + ... + 2,000 = 1,001,000.\]
   
   T  F  Algorithms whose order is \(O(n^2)\) are less efficient than those of order \(O(2^n)\).
   
   T  F  If the set \(A\) contains the element \(x\), then half the subsets of \(A\) contain \(x\).
   
   T  F  \(\text{LCM}(p,q)\text{GCD}(p,q) = pq.\)

2. Let \(\{a_n\}\) and \(\{b_n\}\) be the sequences defined, for \(n \geq 0\), by:
   \[a_n = (n - 1), \text{ and } b_n = (n + 1).\]
   Find \(c_1, c_2, c_3,\) and \(c_4\) when \(c_n = (a_n)(b_n)\).

3. Write out the Euclidean Algorithm and trace its steps to calculate \(\text{GCD}(21,4)\).

4. (a) Give a Recursive Definition for the odd integers.
   
   (b) Find the Big-Oh of the algorithm with complexity:
       \[(n^2 + 1)(4n^3 + 5) + [2n^4 + n(\log n)](3n^2).\]

5. (a) Given \(a = 2^23^25^27^211^913^{11}17^{13}19^{15}23^{17}29^{19}31^{21}\) and \(b = 2^13^25^47^811^{16}13^{32}17^{64}19^{128}23^{256}29^{512}31^{1024}\),
   \[\text{GCD}(a,b) = \] \(a, b\) are Integers with \(a = bq + r\), then \(\text{GCD}(a,b) = \text{GCD}(b,r)\).
   
   (b) List out the search intervals in applying the Binary Search algorithm to find 7 in the list:
   \[0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20\]
   (Hint: Include the mid/test point with the lower interval.)

6. Prove one of the two Theorems below using Mathematical Induction.
   
   \[\text{Theorem 1: For all integers } n \geq 0, \sum_{i=0}^{n} 3^i = \frac{3^{n+1} - 1}{2}.\]

   \[\text{Theorem 2: If } a_0 = 0, a_1 = 10, \text{ and } a_2 = 20, \text{ then } a_n = a_{n-1} + a_{n-2} + a_{n-3} \text{ is divisible by 10, for all } n \geq 3.\]

7. Prove one of the two Theorems below:
   
   \[\text{Theorem 1: If } a, b, p, \text{ and } r \text{ are Integers with } a = bq + r, \text{ then } \text{GCD}(a,b) = \text{GCD}(b,r).\]
   
   \[\text{Theorem 2: The Prime numbers form an infinite set.}\]

8. Prove one of the two Theorems below by Contradiction or Contraposition.
   
   \[\text{Theorem 1: } \sqrt{2} \text{ is irrational.}\]
   
   \[\text{Theorem 2: The Prime numbers form an infinite set.}\]