

CMSC 203 - Discrete Structures - FINAL EXAMINATION - Spring 2006

1. (a) Use the Laws of Logic to show the BICONDITIONAL and EXCLUSIVE-OR are negations of one another.

(b) Find the converse of the following Universal Conditional:

All people who like Mathematics get good jobs.

(c) Use the rules of inference to show the following is a valid argument:

I am rich implies I am happy.

My job is fun or I am not happy.

I am rich.

I do not ride a bicycle implies my job is not fun.

\therefore I ride a bicycle

2. (a) Find $A \times B$ for the sets $A = \{0,1\}$ and $B = \{11, 111, 1111\}$

(b) Using the Properties of Sets, show for any sets A, B and C, $(A \cap B) - C = (A - C) \cap (B - C)$.

3. (a) For $F = \{(1,3), (4,2), (3,7), (7,4), (5,5)\}$, what Domain and Image make F a bijection?

(b) What is the inverse function of F in part (a)?

(c) Find $F(G(x))$ for the following Real-valued functions: $F(x) = 2^{(x+2)}$ and $G(x) = x - 2$.

4. (a) Use the Euclidean Algorithm to find $\text{GCD}(155, 85)$.

(b) Find Big-O of the algorithm whose complexity is $F(n) = (3n^5)(n^6 + 5n^7)(2n^3 + 3n + 2)$.

5. Prove 1 of the 2 Theorems below:

Theorem 1: A set with n elements has 2^n subsets. (Hint: Compare to binary strings)

Theorem 2: The square root of an irrational is irrational.

6. (a) Find the next 3 terms of the sequence $s_n = (2s_{n-1})(s_{n-2}) - (n^2)$ when $s_0 = 1$ and $s_1 = 0$.

(b) Evaluate the series: $\sum_{i=0}^{1000} 4^i - 2i$.

7. Prove 1 of the 2 Theorems that follow by Mathematical Induction.

Theorem 1: If $\Sigma = \{0,1\}$, then $|\Sigma^n| = 2^n$.

Theorem 2: Is $s_n = s_{n-1} + s_{n-2} + s_{n-3} + s_{n-4}$ when $s_0 = s_1 = s_2 = s_3 = 4$ then 4 divides s_n , for all $n \geq 4$.

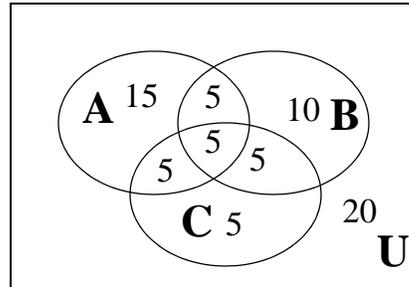
8. (a) If a state issues license plates using the 26 letters $\{A, B, C, \dots, Z\}$, how many distinct plates of 8 letters can they create that begin with C or end with G?

(b) How many plates can they create if they each plate can only have a letter appear no more than once and no plate can contain the string CRASH?

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8. (c) How many distinct piles of 500 coins (pennies, nickels, dimes, quarters, half-dollars, and dollars) can I create from a vast quantity of coins, if I must have at least 50 of each coin in every pile?

9. Consider the following sets with corresponding number of elements indicated in each region:



- (a) Find $P(B \cup C)$
- (b) Find $P(A | (B \cap C))$

10. If UMBC creates a database of information using the fields:
 Name, Address, Phone, Student ID Number, Major, Hours Completed, GPA
 which would likely be Primary Keys?

11. (a) Let $R = \{(a,b) | a,b \in \{0, 1, 2, 3, 4, 5\} \text{ and } a \equiv b \pmod{3}\}$. List the ordered pairs in R.

(b) Let Σ be the alphabet $\{0,1\}$, and define the relation R on Σ^3 to be such that string, s , relates to string, t , if their densities have the same parity. Show R is an Equivalence Relation.

(Note: $\text{Parity}(s) = d(s) \pmod{2}$)

(c) What partition of the Σ^3 does R induce?

12. (a) Find the truth table of the Boolean Polynomial $F(x,y,z) = xyz' + x'z$

(b) Find the Disjunctive Normal Form of the polynomial in (a).