CMSC203 - Spring 2006 - Examination2

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** The sequence {1, –1, 1, –1, 1, –1,...} is an example of an Oscillating Sequence.

T **F** A sequence is just another way to describe a function from the Real numbers to another set.

T **F** A recursive algorithm can always be expressed equivalently as an iterative algorithm.

T **F** $10^{100} = 1 + 9(1 + 10 + 100 + ... + 10^{99})$.

T **F** Functions that are Polynomial order usually grow faster than those that are Exponential order.

T **F** In costing algorithms, the sum of the orders equals the order of the largest term.

T **F** Mathematical Induction is useful for finding new summation formulas.

T **F** Integers that are Relatively Prime have the same Prime numbers as factors.

2. Let $\{a_n\}$ be the sequence defined by: $a_n = n^2 a_{(n-1)}$. Find a_4 when $a_0 = 2$.

3. Fill in the Trace Table of the Division Algorithm when finding (27 DIV 4) and (27 MOD 4).

4. (a) Rank from 1 (least complex) to 10 (most complex) the complexity of algorithms with the following orders:

Order	10^n	$\log n$	$n!$	1	n	n^n	n^{10}
Rank							

(b) Find the Big-Oh of the algorithm with complexity: $(n^4 + n^2)(3n + n^4) + (2n^3 + n^4 + 7)(n^3)$.

5. Use the Euclidean Algorithm to find GCD(99, 21).

6. Prove one of the two Theorems below using Mathematical Induction.

Theorem 1: For all integers $n \geq 0$ and $a \neq 0,1$, $\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

Theorem 2: If $a_0 = a_1 = a_2 = 10$, then $a_n = a_{(n-1)} + a_{(n-2)} + a_{(n-3)}$ is divisible by 10, for $n > 2$.

7. Prove one of the two Theorems below:

Theorem 1: If an Integer divides two other Integers, then it divides any linear combination of the two other Integers. (Definition: For any X and Y, (aX + bY) is a Linear Combination of X and Y, where a and b are arbitrary constants.)

Theorem 2: If a and b are odd Integers, then $(a^3 + b^3)$ is even.

8. Prove one of the two Theorems below by Contradiction.

Theorem 1: If 3 dividing the square of an Integer implies 3 divides the Integer, then $\sqrt{3}$ is irrational .

Theorem 2: No Prime number can divide both an Integer and the Integer's successor.