1. Circle $T$ if the corresponding statement is True or $F$ if it is False.

$T$  $F$  The sequence $\{1, -1, 1, -1, 1, -1, \ldots\}$ is an example of an Oscillating Sequence.

$T$  $F$  A sequence is just another way to describe a function from the Real numbers to another set.

$T$  $F$  A recursive algorithm can always be expressed equivalently as an iterative algorithm.

$T$  $F$  $10^{100} = 1 + 9(1 + 10 + 100 + \ldots + 10^{99})$.

$T$  $F$  Functions that are Polynomial order usually grow faster than those that are Exponential order.

$T$  $F$  In costing algorithms, the sum of the orders equals the order of the largest term.

$T$  $F$  Mathematical Induction is useful for finding new summation formulas.

$T$  $F$  Integers that are Relatively Prime have the same Prime numbers as factors.

2. Let $\{a_n\}$ be the sequence defined by: $a_n = n^2 + a_{(n-1)}$ . Find $a_4$ when $a_0 = 2$.

3. Fill in the Trace Table of the Division Algorithm when finding $(27 \text{ DIV } 4)$ and $(27 \text{ MOD } 4)$.

4. (a) Rank from 1 (least complex) to 10 (most complex) the complexity of algorithms with the following orders:

<table>
<thead>
<tr>
<th>Order</th>
<th>$10^n$</th>
<th>$\log n$</th>
<th>$n!$</th>
<th>$n$</th>
<th>$n^n$</th>
<th>$n^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Find the Big-Oh of the algorithm with complexity: $(n^4 + n^2)(3n + n^4) + (2n^3 + n^4 + 7)(n^3)$.

5. Use the Euclidean Algorithm to find $\text{GCD}(99, 21)$.

6. Prove one of the two Theorems below using Mathematical Induction.

**Theorem 1:** For all integers $n \geq 0$ and $a \neq 0, 1$, $\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

**Theorem 2:** If $a_0 = a_1 = a_2 = 10$, then $a_n = a_{(n-1)} + a_{(n-2)} + a_{(n-3)}$ is divisible by 10, for $n > 2$.

7. Prove one of the two Theorems below:

**Theorem 1:** If an Integer divides two other Integers, then it divides any linear combination of the two other Integers. (*Definition:* For any $X$ and $Y$, $(aX + bY)$ is a Linear Combination of $X$ and $Y$, where $a$ and $b$ are arbitrary constants.)

**Theorem 2:** If $a$ and $b$ are odd Integers, then $(a^3 + b^3)$ is even.

8. Prove one of the two Theorems below by Contradiction.

**Theorem 1:** If 3 dividing the square of an Integer implies 3 divides the Integer, then $\sqrt{3}$ is irrational.

**Theorem 2:** No Prime number can divide both an Integer and the Integer’s successor.