CMSC 203 - Discrete Structures - FINAL EXAMIMATION - Spring 2005

1. (a) Use the Laws of Logic to show: $q \vee \neg (r \rightarrow p) \equiv (q \vee r) \wedge (p \rightarrow q)$
(b) Find the negation of the following Universal Conditional:

All people who like Mathematics get good jobs.

(c) Use the rules of inference to show the following is a valid argument:

$$p \rightarrow q \qquad r \wedge \neg q \qquad p \qquad r \vee s \qquad \therefore s$$

2. (a) Find A × B for the sets A = {x, y} and B = {1, 2, 3}
(b) Using the Properties of Sets, show for any sets A, B and C, (A – B) – C = (A – C) – B.

3. (a) For F = {(b,g), (d,d), (g,a), (n,b), (r,b)}, what Domain and Image make F a function?
(b) Why or why not is the inverse in **(a)** a function?
(c) Find F(G(x)) for the following Real-valued functions: $F(x) = 2^{(2x+1)}$ and $G(x) = 2x - 2$.
4. (a) Use the Euclidean Algorithm to find GCD(154,84).
(b) Find Big-O of the algorithm whose complexity is $F(n) = (3n^5)(n^6 + 5n^4)(2n^3 + 3n + 2)$.
5. Prove 1 of the 2 Theorems below:
Theorem 1: Show that the function f: **R** → **R** given by $f(x) = \pi(x + e)$ is a bijection.
Theorem 2: The square root of 3 is irrational. (Assume the Lemma: If 3 divides INT^2 then 3 divides INT)

6. (a) Find the next 3 terms of the sequence $s_n = 2s_{n-1} - 3s_{n-2}$ when $s_0 = 1$ and $s_1 = 0$.
(b) Find an expression for the series: $\sum_{i=1}^{100} 3(4)^i - 2$.

7. Prove 1 of the 2 Theorems that follow by Mathematical Induction.
Theorem 1: If a set has n elements, then it has 2^n subsets.
Theorem 2: Is $s_n = s_{n-1} + s_{n-2} + s_{n-3} + s_{n-4}$ when $s_0 = s_1 = s_2 = s_3 = 1$ then s_n is even, for all $n \geq 4$.

8. (a) A restaurant serves 4 soups, 6 salads, 8 entrees, 10 desserts, and 12 beverages. How many dinners can they create if each dinner consists of a soup or salad, an entree, and a dessert or beverage?
(b) How many binary strings of length 32 have no more than 3 *zeros*?
(c) How many ways can judges award 1st, 2nd, and 3rd Place prizes to 25 contestants?
(d) How many distinct piles of 200 coins (pennies, nickels, dimes, quarters, half-dollars, and dollars) can I create from a vast quantity of coins, if I must have at least 10 of each in its pile?

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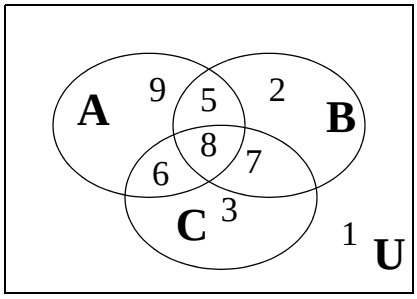
9. Consider the following sets with corresponding number of elements indicated in each region:

(a) Find $P(B \cup C)$

(b) Find $P(A \mid (B \cap C))$



10. (a) Find the inverse relation to

R = {(1,2),(1,3),(2,3),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(5,2),(5,4)}

R^{-1} = _____

(b) Show that the relation $S = \{(a,b) \mid a,b$ are Real and Floor(a) = Floor(b)$\}$ is an Equivalence Relation.

(c) What partition of the Reals does S induce?

11. (a) Find the truth table of the Boolean Polynomial $F(x,y,z) = xy + z$

(b) Find the Disjunctive Normal Form of the polynomial in **(a)**.