Exam 2 Spring 2005 CMSC 203 Discrete Structures

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** The sequence {1, 4, 5, 9, 14, 23, 37,...} satisfies the Fibonacci relation.

T **F** The First (Weak) and Second (Strong) Principles of Mathematical Induction are logically equivalent.

T **F** A recursive algorithm is one which refers to itself.

T **F** 2 + 4 + 6 + 8 + ... + 200 = 10010.

T **F** Functions that are $O(x^{10})$ grow faster than functions that are $O(2^x)$.

T **F** $\sum_{i=0}^{n} (ca_i + b_i) = c\sum_{i=0}^{n} a_i + \sum_{i=0}^{n} b_i$

T **F** In general, Linear Search algorithms are faster than Binary Search algorithms.

T **F** The Euclidean Algorithm can be expressed as both an interative and a recursive algorithm.

2. Let $\{a_n\}$ and $\{b_n\}$ be the sequences defined, for $n \geq 0$, by:

$a_n = n^2 + 2$, and $b_n = (-1)^{(n+1)}$. Find c_0, c_1, c_2, and c_3 when $c_n = (a_n)(b_n)$.

3. Trace the Division Algorithm to find (37 DIV 12) and (37 MOD 12).

4. (a) Rank from 1 (least complex) to 10 (most complex) the complexity of algorithms with the following orders:

Order	n^2	$n\log n$	$n!$	2^n	1	n	n^n	$\log n$	10^n	n^{10}
Rank										

(b) Find the Big-Oh of the algorithm with complexity: $(n^7 + 1)(3n + n^3) + (4n^2 + 2n^3 + 6)(n^6)$.

5. Use the Euclidean Algorithm to find GCD(1060,160).

6. Prove one of the two Theorems below using Mathematical Induction.

<u>Theorem 1</u>: For all integers $n \geq 0$ and $a \neq 0,1$, $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.

<u>Theorem 2</u>: If $a_0 = 2$ and $a_1 = 4$, then $a_n = a_{n-1} + a_{n-2}$ is divisible by 2.

7. Prove one of the two Theorems below:

<u>Theorem 1:</u> The difference of the squares of an integer and its predecessor is odd.

<u>Theorem 2:</u> If a and b are Integers with $a < b$, then there is a Rational, q, such that $a < q < b$.

8. Prove one of the two Theorems below by Contradiction.

<u>Theorem 1:</u> The sum of any rational number and any irrational number is irrational.

<u>Theorem 2:</u> For all Naturals n and primes p, if p divides n, then p does not divide $(n + 1)$.