Exam 1 Spring 2005 CMSC 203 Discrete Structures

1. Circle **T** for True or **F** for False as they apply to the following statements:

T F Every compound statement is either a tautology or a contradiction.
T F The Integers are closed under addition and multiplication.
T F The empty set has no subsets.
T F The set A = {0, 1, 2, ..., 9} has 512 subsets.
T F Disjoint sets have no common elements.
T F The statements $p \to q$ and $\neg p \vee q$ are logically equivalent.
T F The Cartesian Product of two sets is an ordered collection of elements.
T F Bijective functions map sets of the same cardinality to one another.
T F The converse and inverse of a conditional statement are logically equivalent.
T F The argument, *All fish swim* and *Paul cannot swim* therefore *Paul is not a fish* is an example of Universal Modus Tollens.

2. Find the truth table for the compound statement: $r \to \neg[p \vee (q \to \neg r)]$

3. Find the related forms for the Universal Conditional Statement:

Every positive integer that is prime has only 1 and itself as factors.

CONVERSE: INVERSE: CONTRAPOSITIVE: NEGATION:

4. Find set X so that the function, f:**Z** $\to$ X, given by $f(n) = 3n + 5$ is a bijection (one-to-one and onto). X = _____.

5. Show that the function f : **R** $\to$ **R** defined as $f(x) = 3x + 8$ is a bijection.

6. Calculate the following (assuming all strings are from the alphabet {0, 1}):

(a) ℓ(100000000001) **(b)** d(001111110101101)

(c) H(101100011000,111011000000) **(d)** $(\lfloor -3.3 \rfloor + 1)(\lceil 5.9 \rceil)$

7. (a) For the function, f = {(1,4), (2,1), (3,5), (4,3), (5,4)} find $f \circ f \circ f \circ f \circ f$.

$f \circ f \circ f \circ f \circ f$ = {(,), (,), (,), (,), (,)}

(b) Find the Inverse of the function of f.

f^{-1} = {(,), (,), (,), (,), (,)}

8. Use the logic of valid arguments to find the treasure given the following premises:

1. If this house is next to a lake, then the treasure is not in the kitchen.
2. If the tree in the front yard is an elm, then the treasure is in the kitchen.
3. This house is next to a lake.
4. The tree in the front yard is an elm or the treasure is buried under the flagpole.
5. If the tree in the back yard is an oak, then the treasure is in the garage.

9. Use the Properties of Sets to verify for any sets A and B, A = A $\cup$ (A $\cap$ B).