CMSC 203 - Discrete Structures - FINAL EXAMIMATION - Spring 2003

1. (a) Use the Laws of Logic to show: $p \vee (\neg q \rightarrow r) \equiv r \vee (\neg p \rightarrow q)$
(b) Find the negation of the following Universal Conditional:

For all integers, *n*, if *n* is odd, then *n* is not divisible by 2.

(c) Use the rules of inference to show the following is a valid argument:

$$p \rightarrow s$$
$$r \wedge \neg s$$
$$(p \vee t) \rightarrow \neg \mathrm{r}$$
$$\therefore \neg t$$

2. (a) Find A × B for the sets A = {0, 00} and B = {1, 11, 111}
(b) Using the Properties of Sets, show that for any sets A and B, A ∪ (A ∩ B) = A

3. (a) Find the inverse of the function F = {(1,g), (2,d), (3,a), (4,b), (5,b)}.
(b) Why or why not is the inverse in **(a)** a function?
(c) Find F(G(x)) for the following Real-valued functions: $F(x) = 2^{(x+1)}$ and $G(x) = 3x - 5$.
(d) Show that the function f: **R** → **R** given by $f(x) = 3(x + 2)$ is One-To-One and Onto.

4. (a) Use the Euclidean Algorithm to find GCD(444,36).
(b) Find Big-O of the algorithm whose complexity is $F(n) = [n^7(\log^2 n) + n^8 + 6n^3](n^5 + 3)$

5. Prove 1 of the 2 Theorems below:
Theorem 1: For non-zero integers a, b, and c, if (a mod c) = (b mod c), then $a \equiv b$ mod c.
Theorem 2: The square root of 2 is irrational.

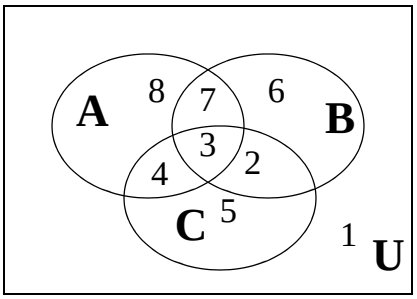
6. (a) Find the next 3 terms of the sequence $s_n = (s_{n-1})(s_{n-2})$ when $s_0 = 1$ and $s_1 = 4$.
(b) Calculate the following series: (i) $\sum_{i=1}^{100} 5^i - 5^{i-1}$ (ii) $\sum_{i=0}^{100} 5^i$

7. Prove 1 of the 2 Theorems that follow by Mathematical Induction.
Theorem 1: If a set has n elements, then it has 2^n subsets.
Theorem 2: Is $s_n = s_{n-1} + s_{n-2} + s_{n-3} + s_{n-4}$ when $s_0 = s_1 = s_2 = s_3 = 1$ then $s_n < 4^n$ for all $n \geq 4$.

8. (a) A restaurant serves 5 soups, 6 salads, 20 entrees, 10 desserts, and 15 beverages. How many dinners can they create if each dinner consists of a soup or salad, an entree, and a dessert or beverage?
(b) How many binary strings of length 8 have: (i) 3 *ones*? (ii) at least seven *zeros*?
(c) (i) How many ways can 6 suspects form a viewing line? (ii) If a certain pair **must** stand next to one another?
(d) How many distinct piles of 100 coins (pennies, nickels, dimes, quarters, half-dollars, and dollars) can I create from a vast quantity of coins, if I must have at least 5 of each in a given pile?

CMSC 203 - Discrete Structures - FINAL EXAMIMATION - Spring 2003

9. Consider the following sets with corresponding number of elements indicated in each region:



(a) Find P(A)

(b) Find P(A | (B ∩ C))

10. (a) Find the inverse relation to

R = {(1,3),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(4,2),(4,5),(5,1)}

R^{-1} = _____

(b) Show that the relation S = {(*a*,*b*) | *a*,*b* are integers and *a* mod 7 = *b* mod 7} is an Equivalence Relation.

(c) What partition of the integers does S induce?

11. (a) Find the truth table of the Boolean Polynomial F(*x*,*y*,*z*) = *x'y* + *xy'z'*

(b) Find the Disjunctive Normal Form of F(*w*,*x*,*y*,*z*) = *x'y* + *xy'z'*