Exam 2  CMSC 203  Discrete Structures  Spring 2003

1. Circle T if the corresponding statement is True or F if it is False.
   T  F  The Fibonacci Sequence is \( \{s_n \mid s_n = s_{n-1} + s_{n-2}, \text{ with } s_0 = 1 \text{ and } s_1 = 1\} \).
   T  F  The First (Weak) and Second (Strong) Principles of Mathematical Induction are logically equivalent.
   T  F  All recursively defined sequences of Integers take on successively larger values.
   T  F  If lazy students fail CMSC203 and Paul passed CMSC203, then we can conclude logically that Paul is not lazy.
   T  F  Functions that are \( O(x^2) \) grow faster than functions that are \( O(2^x) \).
   T  F  The product of a Rational and an Irrational is always Irrational.
   T  F  The product of an Irrational and an Irrational is always Irrational.
   T  F  For every recursive algorithm, there is an equivalent iterative algorithm.

2. Circle V for Valid or I for Invalid with respect to the following arguments:
   V  I  All dogs run fast and Zeke runs slow, therefore Zeke is not a dog.
   V  I  All dogs run fast and Zeke is a dog, therefore Zeke runs fast.
   V  I  All dogs run fast and Zeke is not a dog, therefore Zeke runs fast.
   V  I  All dogs run fast and Zeke runs fast, therefore Zeke is a dog.

3. Let \( \{a_n\} \) and \( \{b_n\} \) be the sequences defined, for \( n \geq 0 \), by: \( a_n = n + 2^n \), \( b_n = (-1)^n \).
   Find \( c_0, c_1, c_2, \) and \( c_3 \) when \( c_n = (a_n)(b_n) \).

4. Rank from 1 (least complex) to 10 (most complex) the complexity of algorithms with the following orders:
   Order \( n^2 \) \( n \log n \) \( n! \) \( 2^n \) \( 1 \) \( n \) \( n^n \) \( \log n \) \( 10^n \) \( n^{10} \)
   Rank

5. Use the Euclidean Algorithm to find \( \text{GCD}(688,124) \).

6. Prove one of the two Theorems below using Mathematical Induction.

   Theorem 1: For all integers \( n \geq 1 \), \( \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \).

   Theorem 2: Every integer greater than 1 is divisible by a prime.

7. Use the Methods of Valid Arguments to obtain the indicated conclusion.
   Premises:
   - Paul does not forfeit his scholarship and Paul goes to class.
   - If Paul does not watch TV, then Paul gets good grades.
   - If Paul watches TV or Paul does not do his homework, then Paul does not go to class.
   - Paul does his homework or Paul forfeits his scholarship.
   Conclusion:
   Therefore, Paul gets good grades.

8. (20 points) Prove one of the two Theorems below by either Contradiction.
   Theorem 1: If every integer has a prime factorization, then the set of primes is infinite.
   Theorem 2: For all integers \( n \) and primes \( p \), if \( p \) divides \( n \), then \( p \) does not divide \( (n + 1) \).