Notation: Let \( \mathbb{R} \) denote the Real Numbers, and \( P(A) \) denote the Power Set of \( A \).

1. (20 pts.) Circle \( T \) if the statement is true or \( F \) if the statement is false.

   - \( T \) \( Z \times Z \subseteq \mathbb{R} \times \mathbb{R} \).
   - \( T \) If \( n \) is an Natural Number, the set \( \{1,2,3,\ldots,n\} \) has \( n^2 \) subsets.
   - \( T \) For any set \( A \), \( \emptyset \subseteq P(A) \) and \( \emptyset \in P(A) \).
   - \( F \) The negation of the statement: All Natural Numbers are even is the statement: Some Natural Numbers are not even.
   - \( T \) \([(36 \text{ DIV } 5) - (93 \text{ MOD } 7)] = 5 \).
   - \( T \) If \( d \mid (x + y) \), then \( d \mid x \) and \( d \mid y \).
   - \( T \) If \( A = \{0,1\} \), then \( A \times A \times A = \{000,001,010,011,100,101,110,111\} \).
   - \( T \) If \( \Sigma = \{0,1\} \), then \( \Sigma^5 = \Sigma \times \Sigma \times \Sigma \times \Sigma \times \Sigma \).
   - \( T \) The set of even integers and the set of odd integers partition the set of integers.
   - \( F \) A conditional statement and its contrapositive are logically equivalent.

2. (6 pts.) Use the Euclidian Algorithm to find \( \text{gcd}(1000,60) \).

3. (10 pts.) Show, without using truth tables, that \( (\neg p \land q) \rightarrow r \equiv \neg p \rightarrow (q \rightarrow r) \).

4. (4 pts.) Give the converse, inverse, contrapositive, and negation of the universal statement: All prime numbers greater than 2 are odd.

5. (10 pts.) Find the Disjunctive Normal Form of a circuit of four inputs in such a way that if the integer value of the inputs is prime, then current flows out of the circuit. (For example, 12 is not prime, and 12 = 1100, so \( f(1100) = 0 \)).

6. (10 pts.) Show the following is a valid argument:

   \[
   \begin{align*}
   &p \rightarrow (q \land r) \\
   \sim r &
   \end{align*}
   \]

   \[
   \therefore \sim p
   \]

7. (40 pts.) Prove 2 of the 4 theorems:

   **Theorem 1**: \( (A \cup B) \cap (A \cup C^c) \cap (B^c \cup C^c) = (A - B) \cup (B - C) \)

   **Theorem 2**: For all integers \( a \) and \( b \), if \( b \) is the successor of \( a \), then \( b^2 - a^2 \) is odd.

   **Theorem 3**: If every integer greater than 1 can be factored as the product of primes, then there is no largest prime.

   **Theorem 4**: If \( a \), \( b \), and \( c \) are integers with \( a = b + c \), then \( \text{gcd}(a,b) = \text{gcd}(b,c) \).