Symbols: $\mathbb{N}$ denotes the Natural Numbers, $\mathbb{Z}$ denotes the Integers, $\mathbb{Q}$ denotes the Rational Numbers, and $\mathbb{R}$ denotes the Real Numbers.

1. Circle T if the statement is true or F if the statement is false.
   T   F   If $A$ is a non-empty set, then $\emptyset$ is a the smallest equivalence relation on $A$.
   T   F   If $A$ is a non-empty set, then $A \times A$ is the largest equivalence relation on $A$.
   T   F   Let $R = (A \times A) \cup (B \times B) \cup (C \times C) \cup (D \times D)$ is an equivalence relation on a set $X$.
         Then the sets, $A$, $B$, $C$, and $D$, partition $X$.
   T   F   One-to-one functions map larger finite sets into smaller finite sets.
   T   F   If a function is onto, its range (co-domain) equals its image.
   T   F   If $f : A \to B$ and $g : B \to A$ are 1-1 and onto functions, then $g \circ f = f \circ g$.
   T   F   If $n$ is a positive integer, $9(1 + 10 + 10^2 + \ldots + 10^{(n-1)}) = (10^n - 9)$.
   T   F   The Weak and Strong Forms of Mathematical Induction are equivalent.
   T   F   If $H$ is the Hamming distance function, then
         $H(111000, 000000) = H(111000, 111111)$.
   T   F   There are as many prime numbers as rational numbers.

2. Rewrite: $\frac{1}{1 \cdot 2} + \frac{2}{2 \cdot 3} + \frac{4}{3 \cdot 4} + \frac{8}{4 \cdot 5} + \frac{16}{5 \cdot 6} + \ldots + \frac{512}{10 \cdot 11}$ as a sum from 6 to 15.

3. Let $R = \{(a,b) \mid a,b \in \{1,2,3,4,5\} \text{ and } a + b \leq 5\}$. Graph $R$.

   \begin{center}
   \begin{tikzpicture}
   \node (2) at (0,0) {2};
   \node (1) at (1,-1) {1};
   \node (4) at (2,-1) {4};
   \node (3) at (1,-2) {3};
   \node (5) at (2,-2) {5};
   \end{tikzpicture}
   \end{center}

4. Prove 1 of the 2 Theorems below by Induction:
   Theorem: A set with $n$ elements has $2^n$ subsets.
   Theorem: Every integer greater than 1 can be factored as the product of prime numbers.

5. Let $f = \{(1,3),(2,1),(3,5),(4,4),(5,2)\}$ and $g = \{(1,2),(2,3),(3,4),(4,5),(5,1)\}$.
   Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 

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6. For the relations graphed below, circle:

- R if it is REFLEXIVE
- S if it is SYMMETRIC
- A if it is ANTI-SYMMETRIC
- T if it is TRANSITIVE
- N if it is NONE of these.

7. Let R be the relation on \( \mathbb{Z} \) given by \( R = \{(a,b) \mid a, b \in \mathbb{Z} \text{ and } a^2 = b^2\} \).

(a) Show R is an equivalence relation on \( \mathbb{Z} \).

(b) Describe the partition of \( \mathbb{Z} \) induced by R.

8. Prove the function \( f: \mathbb{R} \rightarrow \mathbb{R} \) given by \( f(x) = \frac{x + 5}{3} \) is a bijection (one-to-one and onto).