1. Circle T or F as it applies to the associated statement below:

- T F The negation of the statement, “Some integers are positive,” is “Some integers are non-positive.”
- T F The following is a valid argument: T
  \[ \sim q \land p \]
  \[ T \rightarrow (t \lor s) \]
  \[ s \rightarrow \sim p \]
  \[ t \rightarrow r \]
  \[ \therefore r \]
- T F If \( p \equiv q \), then \( p \leftrightarrow q \) is a tautology.
- T F If \( A = \{1,3\} \), \( B = \{1,2,5\} \), and \( U = \{0, 1, 2, 3, 4, 5\} \), then \( (B \cup A)^c = \{3\} \)
- T F If \( A = \{x, y\} \) and \( B = \{a, b\} \), then \( B \times A = \{(a, x), (a, y), (b, x), (b, y)\} \)
- T F If \( f : \{a, b, c, d, e\} \rightarrow \{v, w, x, y, z\} \) is defined as \( f = \{(a,x), (b,z), (c,x), (d,w), (e,y)\} \), then \( f \) is an ONTO function.
- T F If \( f : \{a, b, c, d, e\} \rightarrow \{v, w, x, y, z\} \) is defined as \( f = \{(a,x), (b,z), (c,x), (d,w), (e,y)\} \), then \( f \) is an ONE-TO-ONE function.
- T F If the relation \( R \) on \( A = \{0, 1, 2, 3, 4\} \) is \( R = \{(a, b) \mid a, b \in A \text{ and } b \equiv 4a \mod 5\} \), then \( R = \{(0, 0), (1, 4), (2, 3), (3, 2), (4, 1)\} \)
- T F The relation \( \{(1, 1), (2, 2), (3, 3), (4, 4)\} \) is both SYMMETRIC and ANTI-SYMMETRIC on the set \( \{1, 2, 3, 4\} \).
- T F Let \( S = \{0,1\} \), \( H(s, t) \) be the Hamming Distance Function, and define the equivalence relation \( R = \{(s, t) \mid s, t \in \Sigma^4 \text{ and } H(s, 0000) = H(t, 0000)\} \). Then \( [0011] = \{0011, 0000\} \).
- T F There are \( \frac{14!}{4! \cdot 6! \cdot 4!} \) distinct orderings of the letters \( abbabccacbcabb \).
- T F If \( A, B, \) and \( C \) are sets which partition a set \( X \), then \( |A| = |X| - |B| - |C| \).
- T F If \( n \) and \( r \) are positive integers with \( n \geq r \), then \( P(n, r) = nC(n, r) \).
- T F The Characteristic Polynomial of \( s_n = s_{n-3} + s_{n-5} \) is \( x^5 - x^3 - 1 \).
- T F If a recurrence relation has the General Solution: \( s_n = (A + Bn + Cn^2)(3^n) \), then its Characteristic Polynomial is \( (x - 3)^3 \).

2. Fill in the blanks so the function \( g : \{a, b, c, d\} \rightarrow \{w, x, y, z\} \) is a 1-1 correspondence.
\[
g = \{(a, \_), (b, \_), (c, \_), (d, \_)\}.
\]

3. Find the Boolean Polynomial for a circuit of 5 inputs which outputs a current whenever the first three inputs are the opposite of the last two.

4. How many distinct license plates are there consisting of either 8 non-repeated digits or 3 non-repeated capital letters followed by 5 non-repeated digits?

5. How many different ways can Andrew, Betty, Charles, Diane, Edward, Fay, Gordon, Harriet, Isaac, and June sit around a circular table so that Andrew and Betty never sit next to one another?
6. Show that \( \binom{n}{n - 2} = \frac{n(n - 1)}{2} \).

7. How many integer solutions are there to the equation \( a + b + c + d + e + f + g = 50 \) provided \( a \geq 1, b \geq 2, c \geq 3, d \geq 4, e \geq 5, f \geq 6, \) and \( g \geq 7 \)?

8. Given the recurrence relation \( s_n = 4s_{n-1} + 21s_{n-2} \), what is \( s_{999} \) when \( s_0 = 7 \) and \( s_1 = -1 \)?

9. Prove ONE of the TWO statements below:
   a. If \( d, n, q, \) and \( r \) are integers with \( n = dq + r \), then \( \text{GCD}(n, d) = \text{GCD}(d, r) \).
   b. The square root of 2 is an irrational number.

10. Prove ONE of the TWO statements below:
    a. \( \sum_{i=1}^{n} i^3 = \left( \frac{n(n + 1)}{2} \right)^2 \)
    b. If \( a_1, a_2, a_3, \ldots \) is the sequence: \( a_0 = 3, a_1 = 5, a_2 = 7 \) with \( a_n = a_{n-1} + a_{n-2} + a_{n-3} \), then \( a_n \) is odd for all \( n \geq 3 \).

11. Prove ONE of the TWO statements below:
    a. The function \( f: \mathbb{R} \rightarrow \mathbb{R} \) given by \( 3y + 2x = 6 \) is a bijection.
    b. The relation \( R \) on \( \mathbb{Z} \) given by \( R = \{(a, b) \mid a, b \in \mathbb{Z} \text{ and } b \equiv a \mod 5\} \) is an equivalence relation.