Discrete Structures - Examination 2 - Fall 1996

1. Circle **T** of the corresponding statement is True and **F** if it is False:

T **F** $1 + 2 + 3 + 4 + ... + 500 = 250{,}500$

T **F** The directed graph of a REFLEXIVE relation contains all possible loops.

T **F** If F is a function whose domain and range are the same set, then F is ONTO.

T **F** If $G:A \to B$ is a function with $G = \{(1,-1),(2,-2),(3,-3),(4,-4)\}$, then $\{-1,-2,-3,-4\} \subseteq B$.

T **F** If A is a set, then the relation $R = \{(a,a) \mid a \in A\}$ is the *smallest* Equivalence Relation on A.

T **F** $|\mathbf{Z}| = |\mathbf{Q}|$.

T **F** If A and B are sets with $|A| < |B|$, then there exists a bijective function mapping A to B.

T **F** If $f:A \to B$ and $g:B \to A$ are functions, then $(g \circ f)$ is the *identity* function.

T **F** $1 + 2 + 2^2 + 2^3 + ... + 2^{10{,}000} = 2^{10{,}001} - 1$.

T **F** Let A be a set and R be an Equivalence Relation on A that induces the partition of A into non-empty subsets B and C. If $b \in B$, then the equivalence class, $[b] = B$.

2. Given the bytes 11101100, 00011111, 10111101, and 00100100, which has least Hamming Distance from the byte 11011011?

3. Write $1 - 2 + (3^2) - (4^3) + ... - (12^{13})$ in summation notation ranging from i = 10 to 23.

4. If R is the Equivalence Relation given by $R = \{(a,b) \mid a,b \in \{1,2,...,100\}$ and $a \equiv b \bmod 19\}$, what is [6]?

5. Let R be an Equivalence Relation relation on $A = \{0,1,2,3,4,5\}$ which induces the partition $\{\{0,1\},\{2,3,4,5\}\}$ of A. Draw the directed graph of R.

6. Let $f:\{1,2,3,4,5\} \to \{1,3,5,7,9\}$ be the function $f = \{(1,9),(2,5),(3,7),(4,3),(5,1)\}$ and let $g:\{1,3,5,7,9\} \to \{0,2,4,6,8\}$ be the function $g = \{(1,4),(3,6),(5,2),(7,8),(9,0)\}$. Find $(g \circ f)^{-1}$.

7. Let $\Sigma = \{x,y,z\}$. Find a bijective function to show that Σ^2 and $\Sigma \times \Sigma$ have the same cardinality.

8. Prove 3 of the following 4 statements using the indicated method:

a. Using Strong Induction, show that if *n* is an integer greater than 1 and *n* is not prime, then *n* has a prime factor.

b. Using Mathematical Induction, show that for all integers $a > 1$, $\displaystyle\sum_{i=0}^{n} a^i = \frac{a^{(n+1)} - 1}{a - 1}$.

c. If A is a non-empty set, then A x A is an Equivalence Relation.

d. If $f:A \to B$ is a one-to-one and onto function, then $|A| = |B|$.