1. Circle \( T \) of the corresponding statement is True and \( F \) if it is False:

\[
\begin{array}{ll}
1 + 2 + 3 + 4 + \ldots + 500 = 250,500 & T \\
The directed graph of a REFLEXIVE relation contains all possible loops. & T \\
If \( G:A \to B \) is a function with \( G = \{(1,-1),(2,-2),(3,-3),(4,-4)\} \), then \( \{-1,-2,-3,-4\} \subseteq B \). & T \\
If \( A \) is a set, then the relation \( R = \{(a,a) \mid a \in A\} \) is the smallest Equivalence Relation on \( A \). & T \\
If \( A \) and \( B \) are sets with \(|A| < |B|\), then there exists a bijective function mapping \( A \) to \( B \). & T \\
If \( f:A \to B \) and \( g:B \to A \) are functions, then \( (g \circ f) \) is the identity function. & T \\
1 + 2 + 2^2 + 2^3 + \ldots + 2^{10,000} = 2^{10,001} - 1 & T \\
\end{array}
\]

2. Given the bytes 11101100, 00011111, 10111101, and 00100100, which has least Hamming Distance from the byte 11011011?

3. Write \( 1 - 2 + (3^2) - (4^3) + \ldots - (12^{13}) \) in summation notation ranging from \( i = 10 \) to \( 23 \).

4. If \( R \) is the Equivalence Relation given by \( R = \{(a,b) \mid a,b \in \{1,2,\ldots,100\} \text{ and } a \equiv b \mod 19\} \), what is \([6]\)?

5. Let \( R \) be an Equivalence Relation relation on \( A = \{0,1,2,3,4,5\} \) which induces the partition \( \{\{0,1\},\{2,3,4,5\}\} \) of \( A \). Draw the directed graph of \( R \).

6. Let \( f:\{1,2,3,4,5\} \to \{1,3,5,7,9\} \) be the function \( f = \{(1,9),(2,5),(3,7),(4,3),(5,1)\} \) and let \( g:\{1,3,5,7,9\} \to \{0,2,4,6,8\} \) be the function \( g = \{(1,4),(3,6),(5,2),(7,8),(9,0)\} \). Find \((g \circ f)^{-1}\).

7. Let \( \Sigma = \{x,y,z\} \). Find a bijective function to show that \( \Sigma^2 \) and \( \Sigma \times \Sigma \) have the same cardinality.

8. Prove 3 of the following 4 statements using the indicated method:
   a. Using Strong Induction, show that if \( n \) is an integer greater than 1 and \( n \) is not prime, then \( n \) has a prime factor.

   b. Using Mathematical Induction, show that for all integers \( a > 1 \),
   \[
   \sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}.
   \]

   c. If \( A \) is a non-empty set, then \( A \times A \) is an Equivalence Relation.

   d. If \( f:A \to B \) is a one-to-one and onto function, then \(|A| = |B|\).