CMSC 203 - Discrete Structures Examination 3 - Fall 1994

1. Suppose Maryland produces license plates consisting of 6 characters (digits or letters). How many license plates can Maryland issue if:
(a) each license must have 2 letters followed by 4 digits?
(b) the first and last characters must be of opposite type (either first be a letter and last be a digit or first be a digit and last be a letter)?

2. Suppose I have 4 bags. The first bag has 5 distinct RED balls, the second has 7 distinct YELLOW balls, the third has 9 distinct GREEN balls, and the fourth has 11 distinct BLUE balls.
(a) How many ways are there to choose exactly one ball?
(b) How many ways can I select a RED ball, then a BLUE ball, then a GREEN ball, then a YELLOW ball?
(c) How many ways can I select, in an ordered fashion, 2 balls, the first being BLUE or RED and the second being GREEN or YELLOW?

3. How many ternary words (from the alphabet {0,1,2}) are there of length between 4 and 6, inclusive?

4. How many orderings are there of the letters of the following words:
(a) COMBINE **(b)** REARRANGEMENT

5. Use the *Binomial Theorem* to calculate $(2.01)^5$. (Hint: 2.01 = 2 + .01)

6. Prove that *Pascal's Triangle* works. In other words, verify the identity:

$$\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1} .$$

7. Consider the recurrence relation: $s_n + s_{n-1} - 20s_{n-2} = 0$.
(a) Find its general solution.
(b) Find its particular solution when $s_0 = (-4)$ and $s_1 = 42$.

8. Using the *Iteration Method*, solve the recurrence relation

$$s_n = 5s_{n-1} + 1, \text{ when } s_0 = 1.$$