1. (16 pts.) Construct the truth table for the following statement: $[\sim r \rightarrow (p \lor \sim q)] \lor s$

2. (14 pts.) Prove:
   a. Using truth tables that:
   $\sim(p \rightarrow q) \equiv p \land \sim q$
   b. Without using truth tables that:
   $\sim[\sim(r \land s) \lor (p \land \sim q)] \equiv (r \land s) \rightarrow (p \land \sim q)$
   (hint: use part a)

3. (20 pts.)
   a. Find the truth table of the
      Boolean function $f(x,y) = x + xy$
      \[
      \begin{array}{ccc}
      x & y & f(x,y) \\
      1 & 1 & 1 \\
      1 & 0 & 1 \\
      0 & 1 & 0 \\
      0 & 0 & 0 \\
      \end{array}
      \]
   b. Find the Boolean polynomial whose truth table is:
      \[
      \begin{array}{ccc}
      x & y & z & f(x,y,z) \\
      1 & 1 & 1 & 0 \\
      1 & 1 & 0 & 0 \\
      1 & 0 & 1 & 1 \\
      1 & 0 & 0 & 0 \\
      0 & 1 & 1 & 1 \\
      0 & 1 & 0 & 0 \\
      0 & 0 & 1 & 0 \\
      0 & 0 & 0 & 0 \\
      \end{array}
      \]
   c. Without using truth tables, prove $xy + x'y + x'y' = x' + y$.

4. (10 pts.) Find the circuit diagram for a system of three light switches which turn a light ON only when all three switches are either all ON or all OFF.

5. (20 pts.)
   a. Rewrite the statement: All integers divisible by 2 are even into the form:
      For all ____, if ____, then ____.
   b. Rewrite the statement: Some primes are even into the form:
      There exists ____, such that ____.
   c. Express the statement: "$\forall x \in \mathbb{R}, x^2 \geq 0$" in simple English.
   d. Write the negation of the statement: All Computer Science students hate math.
   e. Write the negation of the statement: $\forall x \in \mathbb{R}, x > 7$ implies $x^2 > 49$.

6. (5 pts.) Disprove by counterexample the statement:
   For all prime numbers, $k$, $(-1)^k < 0$.

7. (15 pts.) Prove the statement: The sum of any even and any odd integer is odd.