CMSC 203 - Discrete Structures - Fall 1993 - Examination 1

1. (16 pts.) Construct the truth table for the following statement: $[\sim r \rightarrow (p \vee \sim q)] \vee s$

2. (14 pts.) Prove:

a. Using truth tables that:

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

b. Without using truth tables that:

$$\sim[\sim(r \wedge s) \vee (p \wedge \sim q)] \equiv (r \wedge s) \rightarrow (p \wedge \sim q)$$

(hint: use part **a**)

3. (20 pts.)

a. Find the truth table of the Boolean function f(x,y) = x + xy

x	y	f(x,y)
1	1	
1	0	
0	1	
0	0	

b. Find the Boolean polynomial whose truth table is:

x	y	z	f(x,y,z)
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

c. Without using truth tables, prove $xy + x'y + x'y' = x' + y$.

4. (10 pts.) Find the circuit diagram for a system of three light switches which turn a light ON only when all three switches are either all ON or all OFF.

5. (20 pts.) **a.** Rewrite the statement: **All integers divisible by 2 are even** into the form:

For all ___, if ___, then ___.

b. Rewrite the statement: **Some primes are even** into the form:

There exists ___, such that ___.

c. Express the statement: "$\forall x \in \mathbf{R}$, $x^2 \geq 0$" in simple English.

d. Write the *negation* of the statement: **All Computer Science students hate math.**

e. Write the *negation* of the statement: $\forall x \in \mathbf{R}$, $x > 7$ implies $x^2 > 49$.

6. (5 pts.) Disprove by counterexample the statement:

For all prime numbers, *k*, $(-1)^k < 0$.

7. (15 pts.) Prove the statement: **The sum of any even and any odd integer is odd.**