

CMSC 203 Discrete Structures Fall 2010 Exam 3 Name _____

1. How many license plates consisting of 7 letters ($\{A, B, C, \dots, Z\}$) or digits ($\{0, 1, \dots, 9\}$) ...
 - (a) ... are 3 letters followed by 4 digits? (b) ... begin OR end with "X1" ?

2. (a) How many ways can line up 8 boys and 8 girls if a certain pair of boys cannot be together?
 (b) How many ways can I order the letters of: HOWMANYWAYS SCANIORDER THE LETTERS OF?

3. (a) How many binary strings of length 7 have an odd number of 1's?
 (b) How many ways can I fill bags of 100 colored marbles if there are 10 colors and I must have at least 5 of each color?
 (c) Show that $C(n+1, k+1) - C(n, k+1) = C(n, k)$

4. (a) A group of 35 students play piano, guitar, or drums, with 26 playing piano, 26 playing guitar, 26 playing drums, 15 playing both piano and guitar, 13 playing both piano and drums, and 17 playing both guitar and drums. What is the probability a student plays all 3 instruments?
 (b) If I roll two 6-sided dice and add the two values, what is the probability that I roll a prime number?
 (c) If I roll two 6-sided dice and add the two values, what is the probability that I roll a prime number given the first die rolls a "3"?
 (d) Determine whether or not I roll a prime number is independent of the first die rolling a "3"?

5. (a) Graph the relation $R = \{(a, b) \mid a, b \in \{1, 2, 3, 4, 5, 6\} \text{ and } (a + b) \text{ is even}\}$.
 (b) The cardinality of the relation that generates the partition $\{1, 3\}, \{4\}, \{2, 5, 6\}$ is _____.
 (c) The cardinality of the smallest REFLEXIVE relation on the set $\{1, 2, 3, 4, 5, 6\}$ is _____.
 (d) The cardinality of the largest TRANSITIVE relation on the set $\{1, 2, 3, 4, 5, 6\}$ is _____.

6. Let $d(s)$ be the density function on binary strings and define the relation $R = \{(a, b) \mid a \text{ and } b \text{ are binary strings of length 4 and } d(a) = d(b)\}$.
 (a) Show that R is REFLEXIVE, SYMMETRIC, and TRANSITIVE.
 (b) Describe the partition of the binary strings of length 4 induced by R.

7. Given the database:

<u>ID</u>	<u>Color</u>	<u>Age</u>	<u>Weight</u>	<u>Length</u>	<u>Condition</u>
100	red	2	2	10	new
101	red	3	2	12	used
102	red	3	3	15	used
103	blue	2	2	11	used
104	blue	3	2	13	new
105	blue	3	3	16	new
200	blue	2	3	20	used
201	green	3	3	23	new
202	green	3	2	25	new
203	green	3	3	26	used
204	green	2	2	30	new

 - (a) What, if any, fields are Primary Keys?
 - (b) Find $P_{2, 3, 6}$ for this instance of the database.

8. (a) Find the DNF of $F(x, y, z) = x'y' + z'$.
 (b) Find the DNF of the Boolean Function with truth table: 0111000100001000