

Fall 2010 Examination 1 CMSC 203

1. Circle **T** for True or **F** for False as they apply to the following statements:

- T F** If a statement is a tautology then its negation is a contradiction.
T F A set with 6 elements has 36 non-empty subsets.
T F The empty set is a subset of all sets.
T F The Range and Image of an Onto function are the same set.
T F If $\Sigma = \{0, 1\}$, then $|\Sigma^6| = 2^6$.
T F The negation of an implication is a disjunction.
T F The densities of a binary string and its negation are equal.
T F $H(00001111, 10101010) = 4$.
T F The converse and inverse of a conditional statement are logically equivalent.
T F The set of all finite-length binary strings is countable.

2. Use the Laws of Logic to show: $p \vee (\neg q \rightarrow r) \equiv r \vee (\neg p \rightarrow q)$

3. Find the negation of the following Universal Conditional: Some people who run fast win races.

4. Use the Rules of Inference to show the following is a valid argument:

$$p \wedge \neg r \quad q \rightarrow r \quad (\neg q \wedge s) \rightarrow \neg p \quad \therefore \neg s$$

5. Sort the set of binary strings of length 4 by their densities, smallest to largest.

6. Using the Properties of Sets, to show $A - (B \cup C) = (A - B) \cap (A - C)$.

7. Given the function $F = \{(1, 2), (2, 1), (3, 2), (4, 1), (5, 2)\}$

(a) What is the Domain of F?

(b) What is the Image of F?

(c) What is the Inverse of F?

(d) Why or why not is the Inverse in (c) a function?

8. Find $F \circ F \circ F \circ F$ for $F: \{0, 1, 2, 3, 4\} \rightarrow \mathbf{R}$ given by $F(x) = 2x - 1$.

9. (8 points) For the given argument, circle **MP** if it is an example of Modus Ponens, **MT** if it is an example of Modus Tollens, **CE** if it is an example of Converse Error, and **IE** if it is an example of Inverse Error.

MP MT CE IE All boys like football and Paul likes football, therefore Paul is a boy.

MP MT CE IE All boys like football and Paul is a boy, therefore Paul likes football.

MP MT CE IE All boys like football and Paul is not a boy, therefore Paul dislikes football.

MP MT CE IE All boys like football and Paul dislikes football, therefore Paul is not a boy.