CMSC 203 - Discrete Structures - FINAL EXAM - Fall 2009 Name___________________________

1. (a) Use the Laws of Logic to show: $(\neg p \to r) \wedge (\neg q \to r) \equiv \neg(p \wedge q) \to r$

(b) Find the negation of the statement: **All, who run fast, win races**.

(c) Use the rules of inference to show the following is a valid argument:

$p \to q \qquad r \to w \qquad \neg q \qquad r \vee s \qquad s \to p \qquad \therefore\ w$

2. (a) Find A × B for the sets A = {10, 11, 01} and B = {100, 010, 001}.

(b) Use the Properties of Sets, to show $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

3. (a) What Domain, Image and Range make F = {(3, 2), (1, 5), (5, 4), (6, 3), (7, 9), (8, 8)} a One-to-One Function?

(b) What is the inverse of F above? (c) (3 pts.) H(01101101, 11011010) = ________.

(d) Give an example of a Bijective (1-1 and onto) Function on {1, 2, 3, 5, 8, 13}

4. (a) Use the Euclidean Algorithm to find GCD(148, 70).

(b) Show the algorithm whose complexity is $(n^6)(2n^7 + 2n^5)(3n^4 + 3n^2 + 1)$ is $O(n^{17})$.

5. (a) Rewrite 2 – 8 + 32 – 128 + 512 – 2048 + 8192 as a summation from 1 to 7.

(b) Find an expression void of summations for the series: $\sum_{i=0}^{n} (3^i + 2i)$.

6. Prove 1 of the 2 Theorems below:

Theorem 1: If a prime number divides the square of an integer, then it divides the integer.

Theorem 2: The square root of a prime integer is irrational. (Assume Theorem 1)

7. Prove 1 of the 2 Theorems that follow by Mathematical Induction.

Theorem 1: There are 2^n binary strings of length n.

Theorem 2: If $s_n = s_{n-1} + s_{n-2} + s_{n-3}$ when $s_0 = 11$, $s_1 = 33$, and $s_2 = 55$, then s_n is odd, for all $n \geq 3$.

8. (a) A restaurant serves 6 soups, 4 salads, 14 entrees, 8 desserts, and 9 beverages. How many dinners can they create if each dinner consists of a soup or salad, an entree, and a dessert or beverage?

(b) How many ways can I choose 5 students from a class of 30 if a certain pair cannot be selected together?

(c) How many ways can judges award 1st, 2nd, 3rd, and 4th Place prizes to 50 contestants?

(d) How many distinct piles of 300 coins (pennies, nickels, dimes, quarters, half-dollars, and dollars) can I create from a vast quantity of coins, if I must have at least 10 of each in its pile?

9. Consider the following sets with corresponding number of elements indicated in each region:

(a) Find $P(A \cup C)$ **(b)** Find $P((A \cap C) \mid (B \cup C))$

(c) Determine whether or not $(A \cap C)$ is Independent of $(B \cup C)$

10. (a) Find $P_{2,3}$ for the database whose records are:

{(1, 2, a), (1, 3, a), (2, 3, b), (3, 1, a), (3, 2, c), (3, 3, b), (3, 4, a), (4, 1, c), (4, 2, c), (5, 2, a), (5, 4, b)}

(b) Show that the relation $S = \{(a,b) \mid a,b$ are Real and $|a| + 1 = |b| + 1\}$ is an Equivalence Relation.

(c) What partition of the Reals does S induce?

11. Find the Truth Table and a Normal Form (either Disjunctive or Conjunctive) of $F(x, y, z) = xy + z'$