

CMSC 203 Discrete Structures Fall 2009 Exam 3

1. How many license plates consisting of 7 letters ($\{A, B, C, \dots, Z\}$) or digits ($\{0, 1, \dots, 9\}$) ...
(a) ... begin AND end with an A, E, I, O, or U?
(b) ... begin OR end with an A, E, I, O, or U?
2. (a) How many ways can line up 10 boys and 12 girls if each gender must be grouped together?
(b) How many ways can I order the digits of the number 31415926535897932384626433832795?
3. (a) How many binary strings of length 12 have at most three 1's?
(b) How many ways can I fill bags of 50 M&M colored candies if there are 8 colors and I must have at least 4 of each color?
(c) Show that $C(n, k) + C(n, k+1) = C(n+1, k+1)$
4. (a) If I roll two 6-sided dice and add the two values, what is the probability that I roll no more than 5?
(b) A group of 35 students play piano, guitar, or drums, with 17 playing piano, 23 playing guitar, 23 playing drums, 12 playing both piano and guitar, 11 playing both piano and drums, and 13 playing both guitar and drums. What is the probability a student plays all 3 instruments?
5. (a) What is the probability that a Natural Number less than 40 is prime given its successor is a power of 2? (Note: Primes less than 40 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37)
(b) Determine whether or not a Natural Number less than 40 is prime is independent of its successor being a power of 2?
6. (a) Graph the relation $R = \{(a, b) \mid a, b \in \{1, 2, 3, 4, 5, 6\} \text{ and } (a + b) \leq 5\}$.
(b) The cardinality of the smallest relation on the set $\{1, 2, 3, 4, 5, 6\}$ is _____.
(c) The cardinality of the smallest REFLEXIVE relation on the set $\{1, 2, 3, 4, 5, 6\}$ is _____.
(d) The cardinality of the largest TRANSITIVE relation on the set $\{1, 2, 3, 4, 5, 6\}$ is _____.
7. Let $f(n) = 3n^2$ and define the relation $R = \{(a, b) \mid a \text{ and } b \text{ are Integers and } f(a) = f(b)\}$.
(a) Show that R is REFLEXIVE, SYMMETRIC, and TRANSITIVE.
(b) Describe the partition of the Integers induced by R.
8. Given the database:
- | Color | Age | Weight | Length | Condition |
|-------|-----|--------|--------|-----------|
| red | 2 | 2 | 10 | new |
| red | 3 | 2 | 12 | used |
| red | 3 | 3 | 10 | used |
| blue | 2 | 2 | 10 | used |
| blue | 3 | 2 | 10 | new |
| blue | 3 | 3 | 12 | new |
| blue | 2 | 3 | 10 | used |
| green | 3 | 3 | 12 | new |
| green | 3 | 2 | 10 | new |
| green | 3 | 3 | 12 | used |
| green | 2 | 2 | 10 | new |
- (a) What, if any, fields are Primary Keys?
(b) Find $P_{2, 3, 5}$ for this of the database.
9. (a) Find the DNF of $F(x, y, z) = xy' + z$.
(b) Find the DNF of the Boolean Function with truth table:
 $F(w, x, y, z) = 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0$